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Advanced mathematical modelling of the temperature distribution and moisture loss in homogenous food during microwave heating

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Advanced mathematical modelling of the temperature distribution and moisture loss in homegenous food during microwave heating

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Abstract

Microwave ovens offer many advantages over conventional heating methods, namely; rapid volumetric heating. The electromagnetic field inside the oven cavity can form standing wave patterns which can lead to hot and cold spots within the food.

Microwave cooking has been studied for many years. The mathematical models that have already been produced use either one dimensional models or full three dimensional simulations. The one dimensional models can highlight key aspects of the heating process, but are limited in their predictive abilities. They are simple models which can offer analytical solutions or fast numerical calculations. Conversely, the three dimensional simulations have excellent predictive abilities for the electromagnetic field in the oven cavity and the heating patterns in the food when compared with experiments. However, these simulations take many hours to run and require careful calibration of parameters.

In this report, we use a variety of analytical and numerical methods, along with experimental data, to produce a simple efficient mathematical model for the heat and moisture changes in the microwave cooking of food. The electric field is approximated using Lambert's law. One dimensional models are employed to analyse the validity of this approximation. These models are then used to investigate the effect of changing dielectric properties on the field, temperature and moisture content of the heated food. A full model is then produced in two dimensions to predict the temperature and moisture content of a rectangular tray of food. This model builds on the analysis and approximations from the one dimensional models, while incorporating essential corrections to account for the three dimensional geometry and field patterns which occur in the oven cavity. The model is then solved numerically and the results were compared with experiments conducted at the CCFRA.

This project is part of a collaboration between the CCFRA, The University of Bath, The University of Greenwich, and Unilever, and is organised through The Smith Institute who manage the Faraday Partnership for Industrial Mathematics. Work is now being carried out to compare the model with a three dimensional simulation of a turntable oven.

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1 Introduction

This report covers the work to date completed on the Microwave project "Heat and Moisture Loss Models in Microwave Cooking" and future work planned. This project is a CASE award PhD award sponsored by the CCFRA and the EPSRC (Engineering and Physical Sciences Research Council.) The aim of the project is to increase the understanding of microwave cooking and develop simple mathematical models to predict the heat and moisture transport in food heating from a chilled state in both stirred and turntable microwave ovens.

1.1 Mathematical Modelling

The aim of this project was to develop relatively simple mathematical models for the microwave heating of chilled foodstuffs and to use these to predict (rapidly) the behaviour of a microwave oven. Although 3-D simulations exist and are able to capture the majority of the physics involved in the heating process, they are not suitable for daily use due to excessive computation time. In order to reduce this computation time for the model to produce accurate results, I have investigated the dominant processes and approximations to these process. In a simple model it is impossible to capture all of the features of the problem. Mathematical modelling and frequent visits to the CCFRA to work with Mr. Greg Hooper and to conduct experiments which have allowed me to quantify and identify the key aspects of the process. Then using approximations and numerical methods I was able to concentrate on the dominant features. This results in a model which produces a close approximation to the actual physical process in a reasonably fast package. In order to speed up the process, I have neglected or approximated certain features of the process to give a model which gives a good approximation to the physical process in a reasonable time.

The problem of modelling microwave heating a foodstuff can be divided into two areas, calculating the field in the food and calculating the temperature and moisture content. In order to produce an efficient model I have made use of the so called Lambert's law approximation to the electric field. In one dimensional problems this approximates the decay of the field, but does not deal with the oscillations which occur due to internal reflections. In order to validate the use of this law I analysed the length scales for which the approximation holds. I also used this approximation to calculate the validity of the temperature calculation.

The dielectric properties of the food change during the heating process and this influences the field and hence the subsequent temperature increase. I was able to develop a mathematical model to highlight the influence of the dielectric changes on the field and temperature distribution in one dimension. This analysis provided a valuable insight into the importance of the changing properties. In particular it was found that the dielectric properties do not vary significantly with temperature for food above 0 degrees Centigrade.

The one dimensional models were all implemented using efficient numerical methods. The one dimensional models as well as providing an excellent insight into the validity and importance of these effects also provided an excellent base with which to extend the model. They were used to produce a full model to predict the temperature and moisture loss of a sample of mashed potato in a rectangular plastic tray. In order to keep the model simple and efficient, the model was then extended to two dimensions and included corrections to allow for the effects of heating at the ends and the corners of the foodstuff and for external field variations. Extending the two dimensional model to predict the moisture loss for a three dimensional sample required an end correction to compensate for the high fields experienced at the ends and corners of the load which are not well approximated using the two dimensional model. The model included the phase change for water boiling and dealt with this using an enthalpy method. The result was a model, which for the rectangular geometry above, proved effective at predicting moisture and temperature in both stirred and turntable ovens.

1.2 Experiments

Throughout the project, I have made frequent visits to the CCFRA. My industrial supervisor Mr. Greg Hooper has provided enormous help in conducting experiments which form an essential part of this report. Mr. Hooper has trained me on the thermal monitoring equipment and has been an invaluable source of advice on all aspects of the project. Through Mr. Hooper's help I have been able to gather information on the heating of mashed potato for three ovens of differing power rating, a 1000W and a 650W mode stirred oven and a 750W turntable oven.

1.3 Faraday Microwave Group

The work contained in this report is part of a collaboration with the CCFRA, the University of Greenwich and Unilever. The microwave project group was organised via The Smith Institute who manage the Faraday partnership for Industrial Mathematics. The ongoing aim of the collaboration is to investigate mathematical models for microwave cooking. The members of the group meet frequently to discuss the progress of each project. The Smith Institute has assigned a technology translator, Dr. Heather Tewkesbury, to oversee the collaboration between the group. Throughout the project we have held regular open meetings with Unilever, Professor Andrew Lacey and also Professor Ricky Metaxas. These meetings have allowed us to keep in close contact with our collaborators and provided guidance for future progress.

The role of each individual member is outlined as follows:

1.4 The University of Bath

The work conducted at Bath, contained in this report, focused on developing simple models for microwave heating of food. The research is sponsored by the CCFRA. Mathematical models for temperature and moisture loss in microwave heating have been developed using both analytical and numerical techniques. Throughout the project frequent visits were made to the CCFRA to conduct experiments and trials. The trials give crucial data to the developments of foods, packaging and heating instructions. A mathematical model able to quickly highlight key aspects of a heating process, would help to guide the researcher in planning experiments. The models were developed to be simple and computationally quick on a desktop PC so as to aid trials at the CCFRA.

1.5 The University of Greenwich

The University of Greenwich, working closely with Unilever developed a full three dimensional electromagnetic field solver. The results from the field solver were coupled with a computational fluid dynamics (CFD) package capable for simulating the heat, moisture and gas flow in the food. The dielectric properties of the food were allowed to change during the heating process and the resulting changes in the field computed. The packages developed at the University of Greenwich presented a full three dimensional simulation of the microwave heating process. These calculations required detailed information about the oven and food load and took many hours to produce results. The team in Greenwich have recently extended their electromagnetic solvers to calculate the fields involved with a turntable oven. It is possible to model a mode stirrer, however, the detail required in the mesh of the metallic vane prohibits any accurate predictions.

The University of Bath and the University of Greenwich teams have worked together to develop and compare mathematical models for microwave cooking.

These models rely heavily on the expertise at the CCFRA to provide input data such as power absorbed by equivalent water loads, required to calibrate models. The CCFRA also provided a wealth of experimental data essential to validating and guiding the mathematical models. The models developed complimented each other very well, the Bath model does not have the full predictive power of the Greenwich simulation, however, the Bath model is able to produce results with reasonable accuracy in a very short time. As part of the collaboration between the two universities and the CCFRA the models have been used to model the microwave heating of food. The experiments conducted at the CCFRA, using the turntable microwave oven were used to calibrate and to validate the numerical models at Bath and Greenwich. Each model is used to calibrate the other, the Greenwich model is able to supply the Bath model with field distributions. The Bath model in turn, highlights the key processes in the heating.

1.6 Report Outline

This report takes the following form:

- *Background and Literature:* A brief summary is given here of the nature of the project and the collaborations formed as part of the microwave group. The work in this project builds on existing models and an overview of these existing models and relevant literature is detailed.

- *One dimensional Models:* The project began by investigating the dominant process involved in microwave cooking. This was achieved by considering simple mathematical models in one dimension. It was found that an approximation can be made to the electromagnetic field using Lambert's law. The dielectric properties of the food are found to change with moisture content, however, for relatively short heating times (less than 5 minute) and when cooking from chilled, these changes are not especially significant. An enthalpy method is used to speed up the numerical calculations to deal with the phase change which occurs at 100 degrees centigrade.

- *Full models:* In this section the full model is presented to model the temperature and moisture lost by a sample of food heated in a microwave oven. This includes extending Lambert's law to two dimensions and adjusting for the effects of high fields experienced at corners and edges.

- *Results and Experiments* The experiments were conducted at the CCFRA using mashed potato in a rectangular CPET plastic container. Thermal imaging and fibre optic thermometry were used to record the temperature evolution of the samples during and after heating. These experimental results were then compared with the results predicted by the model for three different ovens. Two of differing power using a mode stirrer, and a third using a turntable.

The following schematic diagram outlines the relationships and interactions between the models and research teams involved in the microwave project.

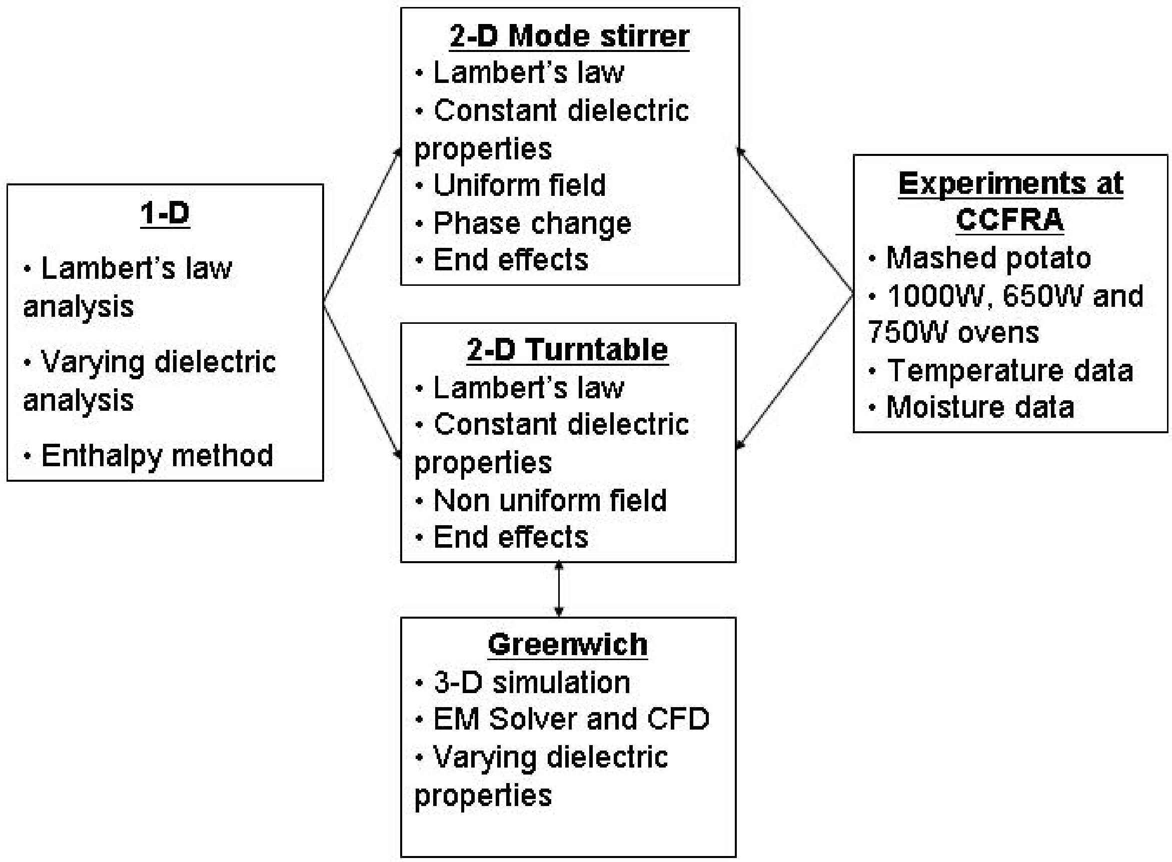

Figure 1: Schematic of the work of the teams

Figure 2: A mode stirred microwave oven showing the food tray and the temperature probes

2 Background

The widespread domestic and commercial use of microwave ovens has led to a significant amount of research into the heating mechanisms involved. Modelling the heating of foodstuffs inside microwave ovens has proved to be a complicated task. The electromagnetic field inside the oven cavity is difficult to measure accurately because any measurement made distorts the field. The field is known to vary a great deal within the oven. The standing wave patterns of the field are of interest to researchers because they lead to variations in the electric field intensity which give hot and cold spots in the food. The electromagnetic field inside the oven is highly dependent on the cavity dimensions and also the dimensions, location and dielectric properties of the load, of which the latter vary during the cooking process. The variability in the oven components and the variety of ovens available also make accurate predictions for the fields inside of a general oven difficult.

A number of reviews of microwave heating have been published recently most notably, Ayappa [2] and Datta [21], who has also published a book [6] on the subject. A more general overview of electric heating applications can be found by Metaxas [16]. This contains details on heating using electromagnetic waves on a range of frequencies.

The one and two dimensional models presented in the literature are particularly useful for highlighting key aspects of the process of heating food by microwaves. They cover a range of situations including investigations into approximations to the electric field such as Lambert's law, the dependency of the dielectric properties on temperature and the influence of the phase change of water at 0 degrees on the dielectric properties and subsequent heating of foodstuffs. Part of our investigation has involved understanding the advantages , disadvantages and limitations of those approximations.

2.1 Lambert's law approximation to the field

One of the main difficulties in modelling microwave cooking mathematically is that of determining the electromagnetic field both inside and outside the foodstuff. Maxwell's equations can be difficult to solve and the continuously changing field patterns require a new field solution to be calculated frequently. Full electromagnetic calculations can take many hours. In order to produce a model which can be computed quickly, I have studied the approximations to the field in order to determine the reliability of these approximations.

A common approximation to the field within a heated sample is to assume an exponential decay of the field intensity at the surface of the load. This is known as Lambert's Law given as follows:

$$\mathbf{E} = E_0 e^{-\beta x} \tag{2.1}$$

where E_0 is the field intensity at the surface and β is the attenuation coefficient which depends on the dielectric properties of the load. This is a measure of the decay of the electric field within a material.

The Lambert law approximation to the electric field can be used to calculate the power absorbed by the food. This is given by a relation in which the power absorbed is proportional to the square of the electric field intensity. This gives the following

$$P = Q_0 e^{-2\beta x}. \tag{2.2}$$

Where P is the power absorbed and Q_0 is the surface power density. This relation gives rise to the power penetration depth, d, which is a measure of the decay of the electric field within the load. The power penetration depth is defined as the distance into the load whereby the power absorbed is 1/e of that at the surface. This gives an inverse relationship between the attenuation constant, β, and penetration depth:

$$d = \frac{1}{2\beta}. \tag{2.3}$$

In a typical foodstuff d is about 1cm. Lambert's law is derived from Maxwell's equations in one dimension [16], and the model is valid for semi infinite domains. The law does not take into account the internal reflections which can occur in shorter sample lengths where the internal reflected waves interfere with the incident waves resulting in a standing wave pattern. The resulting fields take the form of an oscillating electric field intensity centred around an exponential decay [14]. A comparison of Lambert's law and the exact solution to Maxwell's equations is given in the paper by Ayappa et al. [5]. In this paper, Lambert's law is derived along with an analytical solution to Maxwell's equations in one dimension. The two formulations are compared for varying lengths of food. It is found that for short lengths, Lambert's law fails to model the oscillations in the exact solution. For larger domain lengths the amplitude of the oscillations decreases and the approximations agrees with the exact solution. The temperature profiles are found using Lambert's law and Maxwells equations for a material which does not undergo a phase change. We have conducted a similar investigation into the validity of Lambert's law for mashed potato, this model includes a phase change at 100 degrees associated with the boiling point of water. Critically, we find that the length of foodstuff used is just large enough for Lambert's law to be a reasonable approximation. This is a major simplification as it allows us to calculate field in foods without having to use the full Maxwell's equations.

The Lambert law model for the electric field provides a reasonable agreement with experiments on samples with a large lengths. A recent study by Liu, Wang and Sakai into microwave cooking [14] models the thawing of a one dimensional sample of food with temperature dependent properties. This model includes a phase change from frozen to thawed. This is modelled using the enthalpy method which we made use of in our studies to model the phase change through boiling.

Analytical solutions allow us to examine the heating process in more detail. Dolande and Datta [7] outline some temperature profiles common to heating applications. The authors find an analytical solution to the one dimensional heat equation with Lambert's law as a source term. It is found in the early stages of heating, the effect of heat conduction is negligible and the temperature rise is dominated by the local effects of the microwave heating.

The Lambert law model can also be used to model the moisture changes within a heated sample [24] and [17]. The work carried out by Ni [17] provided an extensive moisture transport model for microwave heating in one dimension. Significantly the dielectric properties of a sample are found to be highly dependent on the moisture content. When the electric field is calculated using Lambert's law, the attenuation coefficient in (2.1) has to be calculated taking into account the moisture content of the sample. It was found that the field was able to penetrate further into a dry sample than a wet one, because as the moisture content decreases, the materials ability to absorb microwave energy also decreases. This leads to a change in the electric field and the power absorbed by the sample. This will have an effect on the temperature and moisture content of the load. This effect was investigated as part of my project and will be addressed in the following section.

2.2 2-D models

Lambert's law is derived from Maxwell's equations in one dimension, however it can be extended to higher dimensions. In the 1994 paper by Zeng and Faghri [25], a two dimensional model microwave cooking was presented. Extending Lambert's law into two dimensions the model predicted the power absorbed by a cylindrical sample. The temperature increase of the sample was calculated beginning in a frozen state. The phase change from solid to liquid was modelled using the enthalpy method. The model assumed that the attenuation coefficient (derived from the dielectric properties of the sample) were temperature dependent. The model was calibrated using the power absorbed by a water load. The equations were solved using a finite volume technique and the results compared with experiments conducted on Tylose samples. The predicted temperatures were found to have a good agreement with experiments. The predicted thawing times were also found to be consistent with the experimental data.

In reality the field inside a two dimensional material is more complicated than the exponential model. The field inside a sample with constant dielectric properties obeys Helmholtz equation [16]. In a paper by Ayappa, a two dimensional model for microwave heating was developed solving the Helmholtz equation in two dimensions for long cylindrical and square rods.

2.3 3-D Simulation

The experiments were conducted on real three dimensional objects clearly to understand these completely, we must consider the existing three dimensional simulations of microwave heating. These simulations provide an accurate overview of all the physical processes involved in microwave heating, however, they are derived from extremely complicated mathematical models and generally require a large amount of parameters to describe the physical processes involved; the values of which are not well known and often have to be calibrated following experiments. The complexity of the models results in large computation times even on high performance computers, often taking many hours to run. This is the penalty one must pay for the accuracy in field, temperature and moisture data. This information is essential in certain areas providing invaluable insight into the electromagnetic fields inside ovens. These models however are unsuitable for everyday use where calculations are required daily for several samples. We began our survey of literature with the work of Metaxas' group at Cambridge University. The Electricity Utilisation Group which is part of the engineering department at Cambridge University, have extensive experience in modelling electromagnetic fields in oven cavities. In the process of this microwave project a meeting was arranged with Metaxas, the CCFRA, Greenwich University and Bath University to make use of the extensive experience of Metaxas group in the area of field calculations.

In their 2003 paper 'Finite Element Time Domain Analysis of Microwave Heating Applications' [9], Metaxas and Hallac outlined a finite element solution to the microwave field inside a microwave oven cavity. Using finite element methods the electromagnetic field was calculated in a microwave oven with a rectangular rubber load. The numerical calculations converged after several hours of calculation time to reveal an excellent agreement with experimental values for the reflection coefficient.

A combined electromagnetic and thermal finite difference time domain model for microwave heating is presented in the 1995 paper by Ma et al. [15]. The electromagnetic field is coupled with the thermal properties of the sample by the dielectric properties which are temperature dependent. The temperature of the sample was found by solving the heat equation with a convective heat loss boundary. The model was compared with experiments conducted on a phantom food gel with similar properties to meat. The model simulated the heating of a chilled sample initially at 5 degrees centigrade. The model found good agreement with thermal images and reproduces a hot spot within the sample.

2.4 Conclusion

The one dimensional models investigated in the literature examine many important aspects of microwave heating, from field approximations to phase changes during thawing. These models are generally applied to experiments on large samples of food where the one dimensional approximation for food is valid. The three dimensional simulations, although able to simulate the field, heat and moisture changes, require calibration and are sensitive to a number of parameters. They compare very well with experiments however they can take many hours to compute a solution.

The work conducted on this project aimed to extend the one dimensional models to two dimensional ones which could be used to calculate the temperature and moisture loss of typical samples sizes and shapes used in microwave cooking. The models were validated against experiments and three dimensional simulations.

3 A one dimensional model for heating in a microwave oven

In this section we describe our investigation into models for the heating of foodstuffs in a microwave oven in one dimension. Whilst real foodstuffs are not one dimensional, this investigation was useful as it allowed us:

- To look at the validity of Lambert's law approximation to the electric field inside the food and then to the resulting temperature distribution
- To look at the effects of dielectric variation on the resulting temperature distribution
- To develop fast numerical methods for calculating the temperature and moisture content based on enthalpy methods.

We extended the work by Ayappa and Liu to derive the Lambert's Law approximation from Maxwell's equations for electromagnetic waves. These equations were then solved in one dimension to find an exact solution for the electric field intensity inside a material exposed to an electromagnetic wave on one side. This enabled me to compare the Lambert's law approximation to the field with the exact solution in one dimension. An investigation was carried out for the power absorbed by various lengths of sample (in the case of mashed potato.) The heat equation was solved numerically to calculate the temperature profiles for each of the two formulations, exact and Lambert's law. This model for the heating of food included a simple phase change at 100 degrees, this phase change assumed that at 100 degrees bound water in the food boils and all energy at that point is used to vapourise the remaining water. This formulation is known as the Enthalpy method and is a well understood and widely used method for phase change problems. The dielectric properties of the food material changes during the heating process. The changing temperature, moisture content and salinity can all have an influence on the dielectric properties of the food. We investigated the effects the changing properties have on the evolution of temperature, moisture content and the electric field inside the food.

3.1 Validity of Lambert's Law

Following Ayappa [5] we examined the validity of Lambert's law in terms of the typical length scales of food. The power absorbed by a one dimensional sample of food (in the particular case for mashed potato) was found exactly using Maxwell's equations. This field was also approximated using Lambert's law. A comparison was carried out to investigate conditions for Lambert's law to approximate the exact power absorbed in one dimension. We began by varying the length of the sample and plotting the resulting profiles for the power absorbed using the two formulations. This gave me a valuable insight into the length scales for which the Lambert's law provided a reasonable approximation to the exact field in the food.

3.1.1 Surface power density

In order to examine the power absorbed profiles, I needed to estimate the surface power density. This value depends on the power rating of the oven, the dielectric properties of the food and also the dimensions of the load. The surface power density was found by integrating the equations determined for the power absorbed in the sample over the total sample volume and comparing this value with the power rating of the oven. The profiles in Figure 3 were calculated for both Maxwell's equations and Lambert's law for varying lengths, L, of the food.

It is clear to see that for large lengths, L, Lambert's law is a good approximation to the exact solution of Maxwell's equations. For smaller lengths the solutions to Maxwell's equations oscillates considerably. The oscillations in the exact solution are the results of internal reflections inside the food. These reflected waves interfere with the transmitted wave resulting in the standing wave patterns observed above. As the length of domain increases the amplitude of the oscillations decreases. This is due to the decay of the transmitted wave inside the material which results in a reflected wave of reduced intensity. The typical lengths of sample which are investigated in this report have a smallest length of around 2cm.

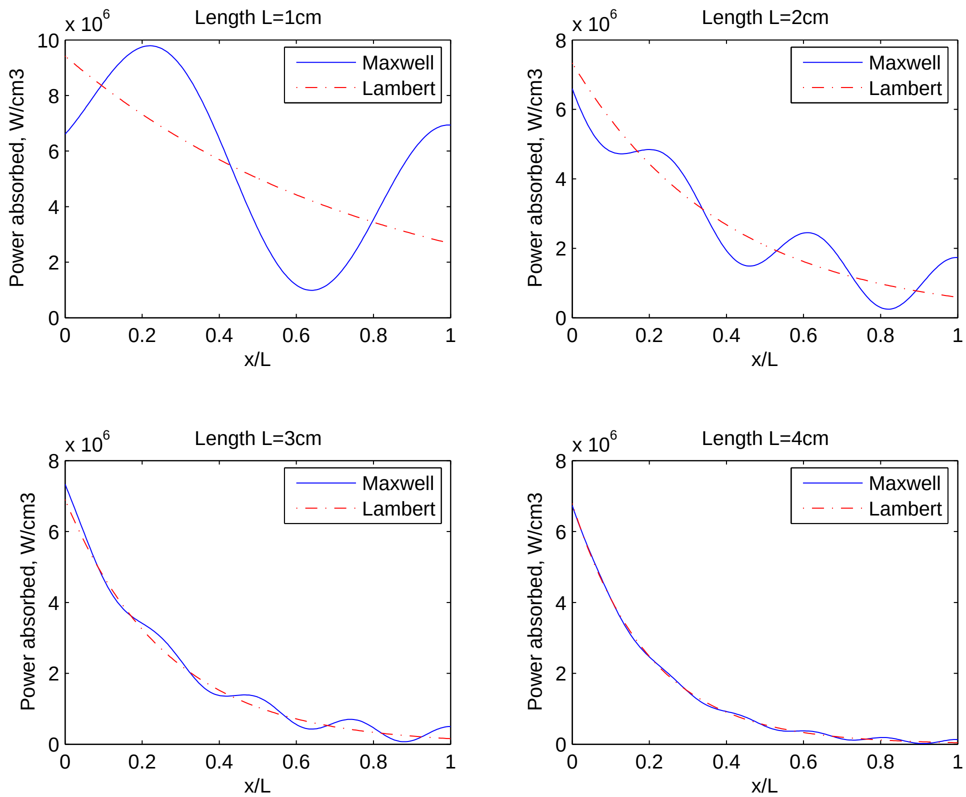

Figure 3: A comparison of the Lambert law approximation of the power absorbed derived from solutions to Maxwell's equations

3.2 Conclusion on the validity of Lambert's law

It was found that Lambert's law is a good approximation to the field inside the food if the length of food is sufficiently large, in the case of mashed potato this length should be greater than 2cm. Lambert's law does not approximate the oscillations which occur in lengths smaller than 2cm.

3.3 The temperature profile

I then investigated the effects these two fields, the exact and Lambert's law have on the temperature profile in the foodstuff assuming initially constant dielectric properties. Using the two expressions for the power absorbed as a source term in the heat equation, the temperature rise in samples of mashed potato were computed. An enthalpy method formulation was used to account for the phase change of water at 100 degrees centigrade. The enthalpy method is a very fast way of dealing with phase change problems and will be described in section 3.8. Unlike other methods, there is no need to solve equations across the phase change front.

The phase change was modelled by solving the specific heats of the system, and calculating the temperature from this information. The heat equation required boundary conditions to allow for heat losses at the surface, which were imposed at the surfaces of the food. At one face a convective heat loss was imposed, at the opposite face I imposed an insulated condition.

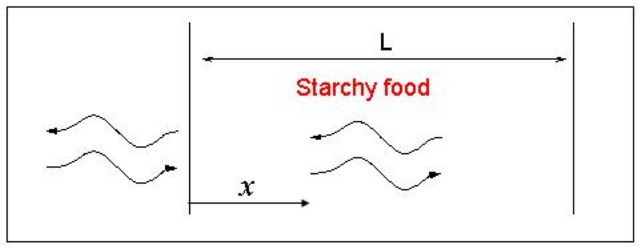

Figure 4: Schematic showing the microwave heating of a one dimensional sample of food of length L

3.4 The heat equation

The heat equation describes the change in temperature of a material with heat transfer through conduction. In the case of the foods under consideration convection was negligible. To describe the temperature of food as it is heated in a microwave oven we solved the following:

$$\rho \frac{\partial u}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + P \tag{3.1}$$

The depth into the food is given by $x > 0$ (where $x = 0$ is the surface), u represents the enthalpy of the food and P represents the power absorbed by the food from the electromagnetic field, we used both the exact solution of Maxwell's equations and also the approximation due to Lambert's law. In this equation ρ

is the density of the food and k is the thermal conductivity of the food. The principle mode of heating in the microwave heating of foods is through dipole alignment of the polar water molecules. We assume that at 100 degrees centigrade all the energy is used to vapourise the bound water in the material. We write the enthalpy, u, in terms of the latent heat of the material:

$$T = \frac{u}{c} \quad T \leq 100 \tag{3.2}$$

$$T = \frac{u(100)}{c} \quad T \geq 100 \tag{3.3}$$

where c is the specific heat capacity of the food.

We applied the following boundary condition at $x = 0$

$$k\frac{\partial T}{\partial x} = h_c(T - T_a) \tag{3.4}$$

where h_c is the convective heat transfer coefficient. This boundary condition represents heat loss at the surface of the food through convection. At $x = L$ we applied a boundary condition to model an insulated surface given by

$$\frac{\partial T}{\partial x} = 0. \tag{3.5}$$

The heat equation with the two power terms above and using the convective heat loss boundary condition were then solved numerically. The heat equation was discretised in space with the finite difference scheme. This leads to a large system of stiff ordinary differential equations to be solved to find the temperature. These in turn were solved by using the fast and accurate gear solver ode15s in the general purpose package Matlab to produce the solution in space and time. We again plotted the temperature profiles related to the two profiles of absorbed power and as above investigated the effect of increasing the length of food sample.

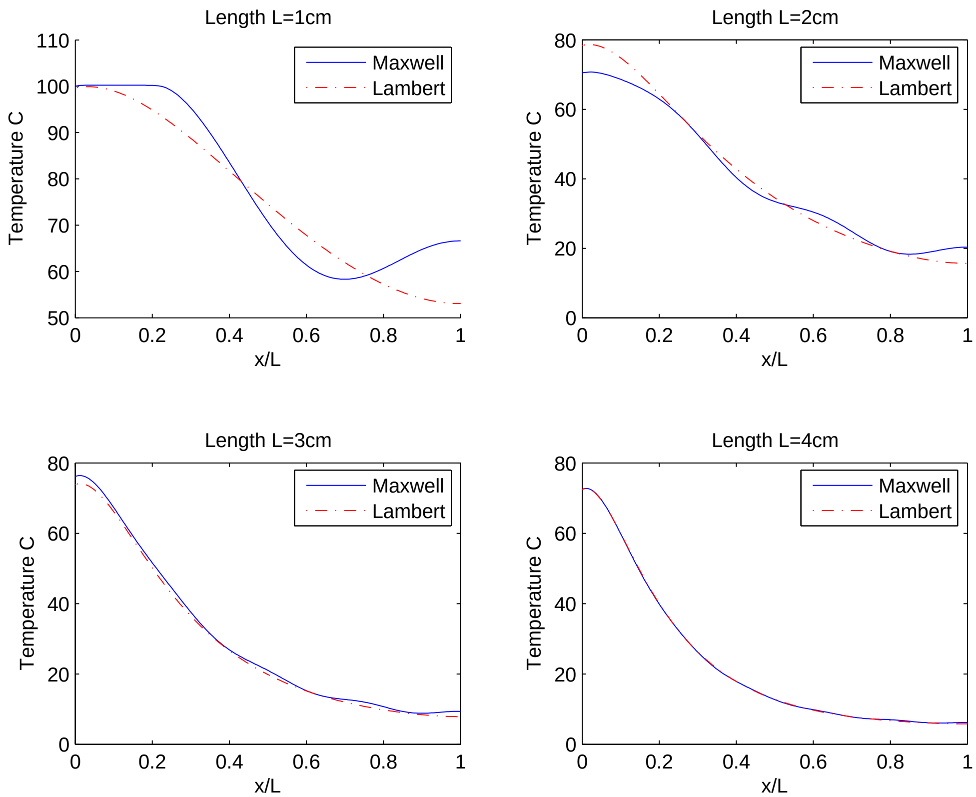

Figure 5: Comparison of the temperature distributions at t=60 seconds using the power absorbed derived from both the solutions to Maxwell's equations and the Lambert Law approximations

It is clear from figure 5 that the magnitudes of oscillations apparent in the absorbed power profiles were reduced in the temperature profile. In the smallest length case considered here with variation is clear this becomes less so even in the case of 2cm sample length. This is due to the averaging effect of the heat equation smoothing out field variations in the source term.

3.5 Conclusion on Lambert's law and heating

Lambert's law is a good approximation to the field in one dimension for lengths greater than 2cm and the averaging effect of the heat equation makes the temperature approximation give an even closer value to the true solution.

This analysis provided an excellent basis to extend the model into two dimensions. The analysis of the validity of lambert's law provided an estimate of the length scales for which such a model is valid. The temperature calculation provided a basis for a fast numerical scheme. The enthalpy formulation provided an excellent way of dealing with the phase change and also results in a much faster calculation.

3.6 Dielectric sensitivity

As mentioned earlier, the dielectric properties are sensitive to many factors including phase changes (water absorbs more energy than ice in a microwave oven), temperature, moisture content and also salinity. This project was concerned with the heating of chilled foods, and so there is no phase change from frozen. However, the temperature and moisture dependence needed to be addressed. This led to an investigation into the role that the changing dielectric parameters have on the field inside the food and the resultant temperature profiles they induce. I made use of published experimental data [20],[19] to ascertain the dependency of these parameters on temperature and moisture content. Using this data, I was then able to construct a one dimensional model for heat and moisture loss with changing dielectric parameters. The field was recalculated at each time step and coupled to the heat equation through the moisture dependent dielectric properties.

In order to investigate only effects of the changing dielectric properties, I assumed that there were no internal reflections in the food. This eliminated the oscillatory solutions investigated previously. In order to calculate the electric field intensity as moisture and temperature vary, it became essential to obtain data on how the dielectric properties change. Experimental literature gave tables of these values for a variety of materials, temperatures and moisture contents. Regier et al. ([20]), measured the dielectric properties of mashed potato for a variety of temperatures in a 2.45GHz microwave oven using a cavity perturbation technique. A marked change is observed in the dielectric properties as the food passes from a frozen phase to a liquid phase at 0 degrees. This is partly due to the fact that ice absorbs significantly less electromagnetic energy than water. In the liquid phase it was found that the dielectric properties remain constant with temperature. This eliminated the temperature variation in the dielectric properties however as the food reaches 100 degrees and bound water is boiled off the changing moisture content strongly influences the dielectric properties. In the book Engineering Properties of Food ([19]), Mudgett outlined the methods of finding the dielectric properties of a material. The author gave the experimental values for the dielectric constant, ϵ' , and dielectric loss, ϵ'', for mashed potato with varying moisture contents. The values given were for heating at a microwave frequency of 3GHz. Taking values from the data given in the book, I constructed a fifth order polynomial approximation to the data to calculate the dielectric properties at all temperatures in the range 0-120 degrees rather than just the at the temperature data points given in the literature. This was essential for calculating the electric field intensity as I will outline later. I arrived at the figure 6 giving the dielectric properties with changing moisture contents.

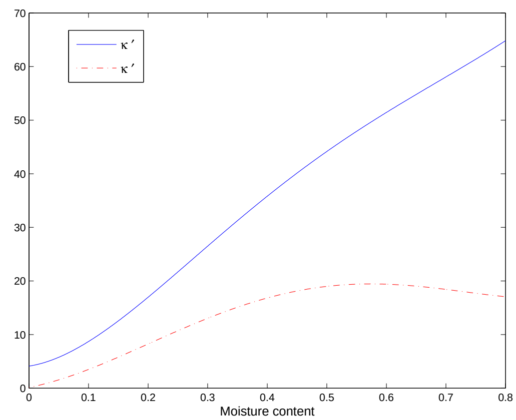

Figure 6: Variation of the dielectric properties of mashed potato at 3Ghz as a function of moisture content

As the moisture content of the load change so does the dielectric properties of the food. It is this drop which alters the field inside the load. The dielectric properties are often combined to form the power penetration depth, d. The experimental data allowed us to determine the relationship between penetration depth and moisture content as given in figure 7.

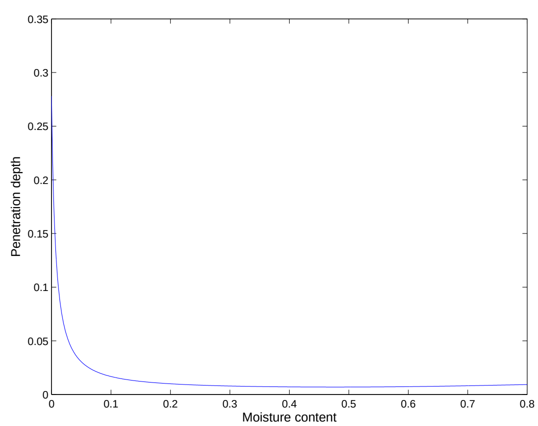

Figure 7: Variation of the penetration depth of microwaves in mashed potato at 3Ghz with changing moisture content

It is clear from the above figure, that as the moisture content of the food decreases the penetration depth of the sample increases. In the early stages of heating there is little change in the moisture content of the food and so we should expect that the field will remain unchanged. In the later stages of heating as moisture is boiled off, we observe the moisture content of the food dropping to 20%, and it is in this period that we observe the most significant effect on the field.

3.7 Heat and moisture model

We can now demonstrate the impact of the changing dielectric properties on the heat and moisture content of the food. The temperature of the food was calculated using the heat equation as described in the previous section. The moisture loss was calculated using the assumption that at 100 degrees centigrade all energy at that point is used to vapourise the bound water in the sample. This gave us the necessary data to update the electric field intensity.

We began with the heat equation and the enthalpy method describing conduction within the material (3.1) and (3.3. At the boundaries we applied a convective heat loss condition at the face exposed to the microwave field (3.4) and an insulating condition at the opposite face (3.5). The moisture loss was described using the latent heat of vapourisation and is proportional to the power absorbed and is given by

$$M_t = -\frac{P}{\lambda}. \tag{3.6}$$

The moisture content of the material was found by solving (3.1)-(3.5) together with (3.6). Whereas in section (3.4) P was regarded as constant, in this case P itself was updated to allow for the varying dielectric properties as a function of the moisture content, M. In particular the local dielectric properties were found from Figure (6) and using these the total field and power estimated from the power rating of the oven. The numerical computations revealed the temperature, moisture and field values presented in Figures (8)-(11)

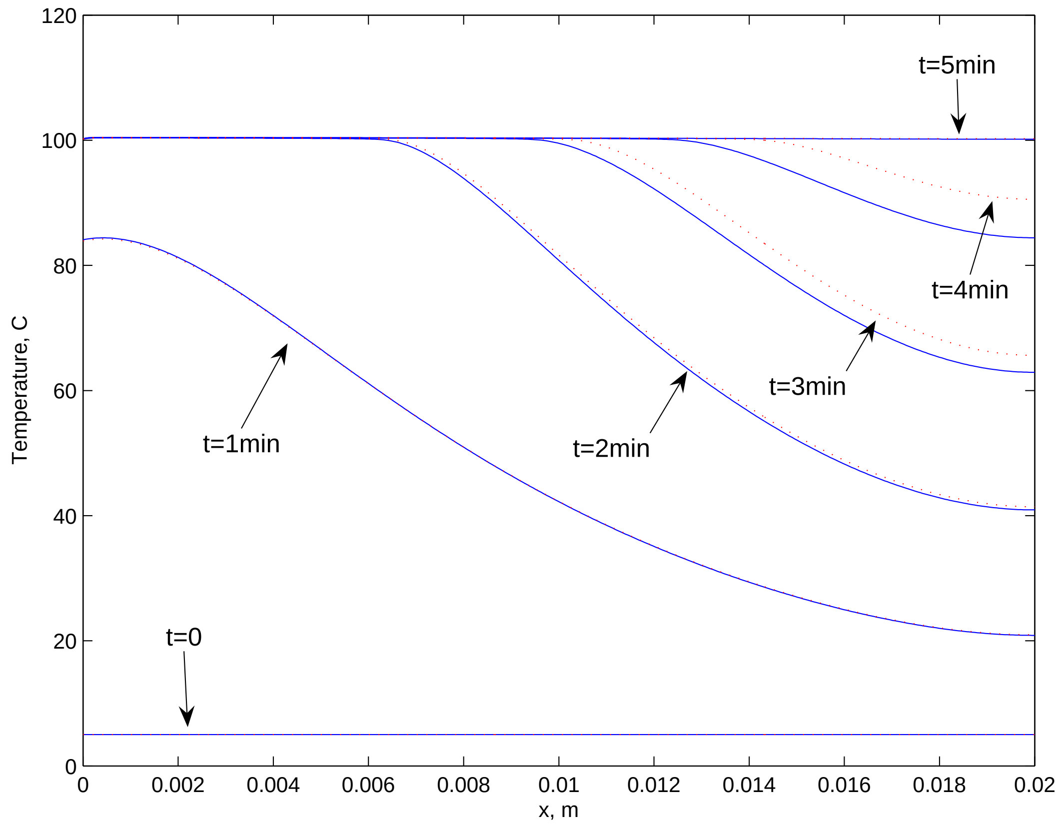

Figure 8: Temperature profiles of a 1-D sample of mashed potato of length L=2cm heated in a microwave oven for constant and varying dielectric properties. Constant dielectric properties in red dash, varying dielectric properties in solid blue

It is clear from (8) that the temperature rise is most prominent at the surface of the food. Where the material is exposed to the microwaves, there is also a heat flux out of the material at this surface due to convective heat loss. The penetration depth of the electromagnetic waves used in the heating is of the order of 1cm and so we see the heat rise from conduction from the surface boundary layer. Once the material reaches the boiling point of water we start to lose moisture. We observe that at 5 minutes heating the entire sample has reached 100 degrees centigrade and the entire sample experiences moisture loss. The moisture loss model predicts the moisture loss profiles presented in (9).

The solid blue lines represent the temperature increase with moisture dependent dielectric properties. The dashed red line represent constant dielectric properties. In the early stages there is no moisture loss as the temperature is below boiling and the two calculations overlap. In the later stages we see the influence the changing dielectric properties have on the temperature. However this is only noticeable briefly before the entire sample reaches 100 degrees centigrade.

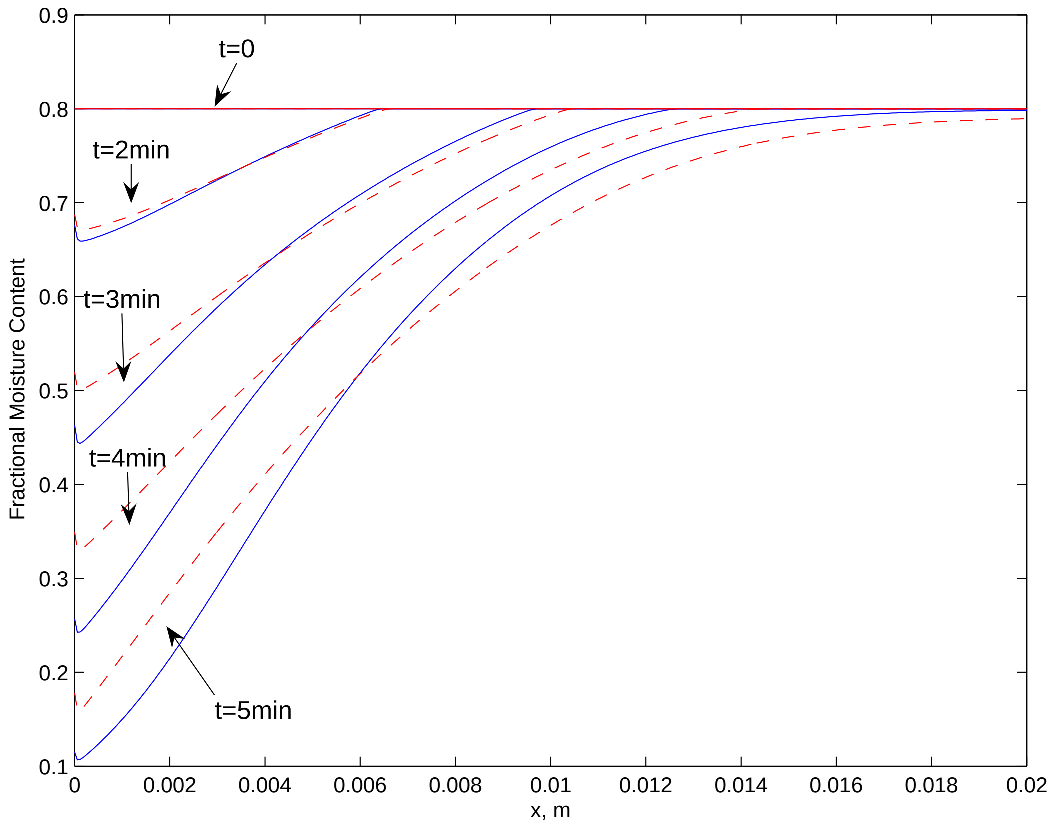

Figure 9: Moisture content profiles of a 1-D sample of mashed potato heated in a microwave oven for constant and varying dielectric properties. Constant dielectric properties in red dash, varying dielectric properties in solid blue

In the early stages of heating there is no change in the moisture content and the field will not be affected however as the moisture is lost near the surface the field begins to change. As the moisture content of the sample decreases the dielectric properties of the food decrease, increasing the penetration depth of the food.

Again we see the blue solid lines representing moisture dependent dielectric properties agree with the red dashed lines representing constant dielectric properties in the early stages of heating. The lines diverge after 2 minutes heating and continue to do so after 5 minutes heating. The changing properties influence the electric field intensity which in turn influences the rate of moisture loss. After 5 minutes heating we see a difference of only 5%.

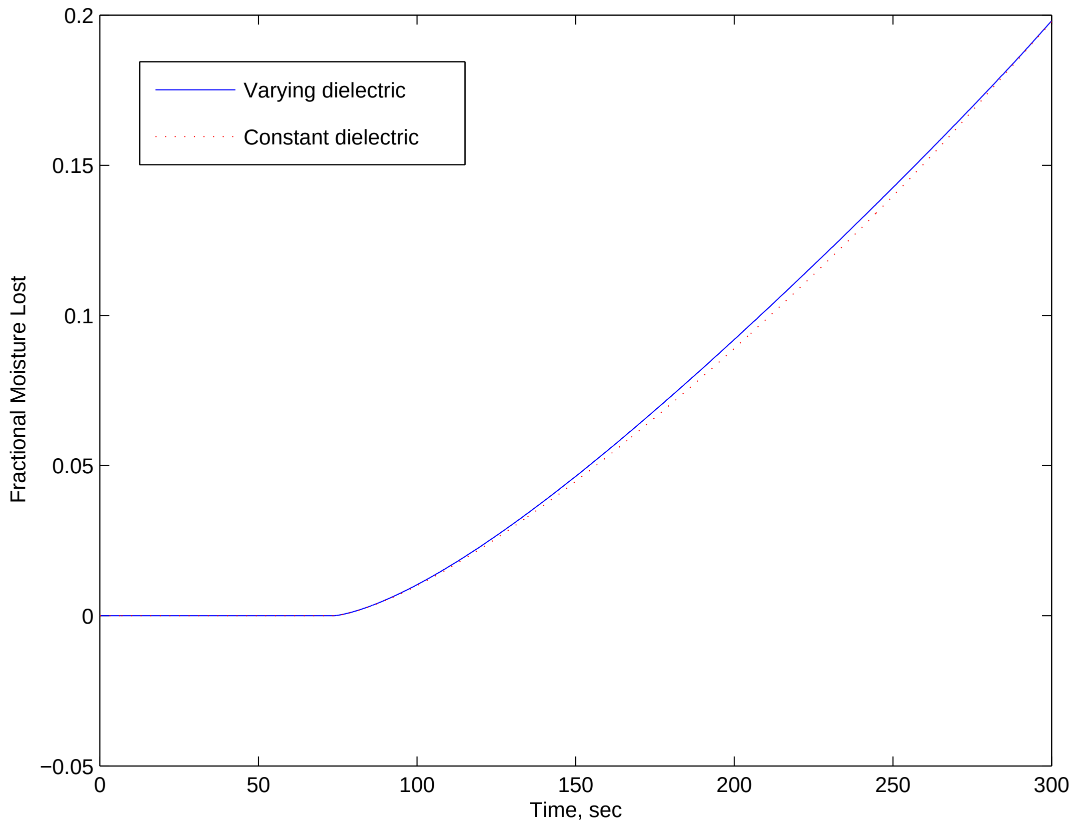

Figure 10: Moisture lost from a 1-D sample of mashed potato heated in a microwave oven for constant and varying dielectric properties. Constant dielectric properties in red dash, varying dielectric properties in solid blue

We present in figure (10) the moisture lost from the sample with time. As in the previous figure (9) we see that in the early stages of heating there is no change in the moisture content. The figure shows the moisture lost for the whole sample over a heating time of 5 minutes using constant dielectric properties in red dash and moisture dependent dielectric properties in solid blue, there is little difference between the two curves.

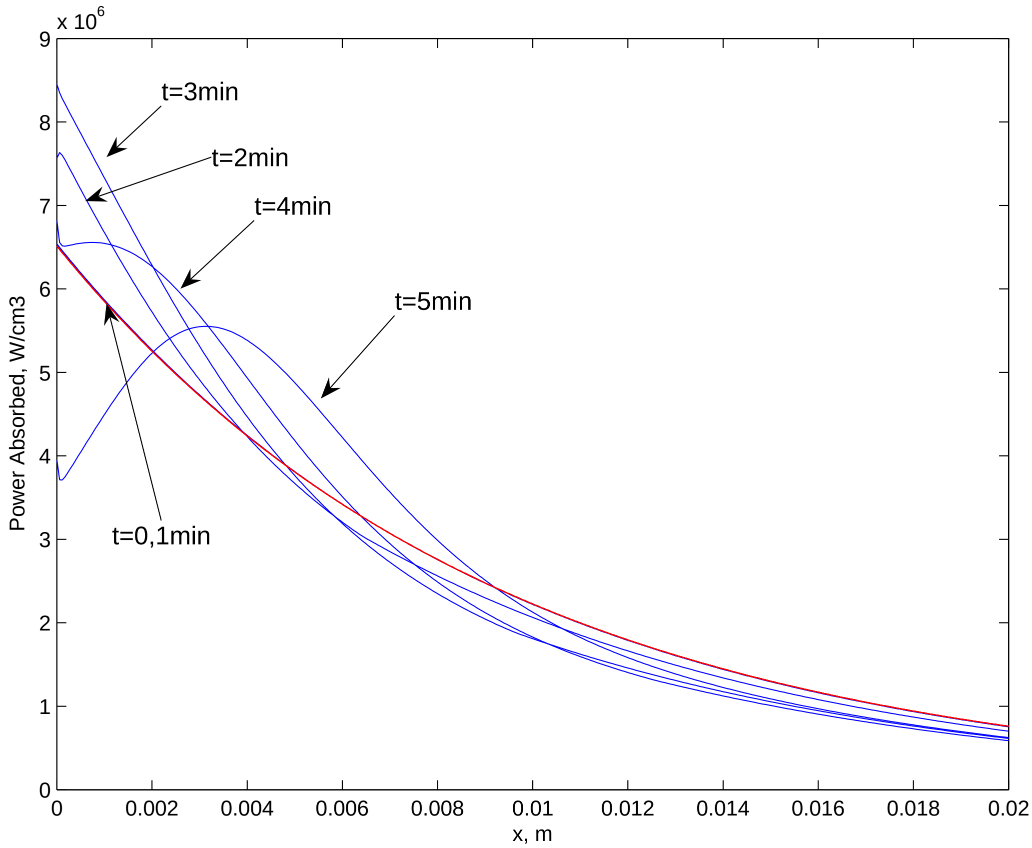

Figure 11: Power density profiles of a 1-D sample of mashed potato heated in a microwave oven for constant and varying dielectric properties. Constant dielectric properties in red dash, varying dielectric properties in solid blue

In figure (11) we present the change in the absorbed power over the heating process. Initially, we have the exponential decay of the surface power density predicted by Lambert's law. The field remains unchanged for the next minute, after two minutes heating the field decays more rapidly. This corresponds to a decrease in the penetration depth as the moisture content decreases because only in the later stages does the dielectric loss (which has the greater influence on the penetration depth) change. In the later stages of heating where the moisture content is significantly lower than the initial value we observe a marked change in the field. The low moisture content at the surface results in an increase in the penetration depth. This means that the field is less attenuated by the load and so is absorbed less in the surface boundary layer. Thus we see that at 5 minutes heating the absorbed power increases into the sample before decaying as the field enters a region with higher dielectric properties.

3.8 The Enthalpy method for the numerical calculations

Experiments confirm that by far the greater majority of the moisture loss occurs after the sample reaches 100 degrees centigrade. It is at this temperature that all energy absorbed by the sample is used in vapourising the liquid rather than increasing the temperature of the fluid. It is in this period that we focus our attention to calculating the moisture lost during heating. There exists a large amount of literature regarding phase change problems. One method of solution is to formulate the model as a Stefan problem. The solution of this problem is to consider it as a moving boundary problem where the phase change front is tracked and appropriate boundary conditions applied across the front. This method of solution is computationally expensive. The purpose of this project was to introduce a simple model for the heat and moisture transfer in microwave cooking which could be implemented in a fast and reliable numerical method and so we turned our attention to the enthalpy method. The enthalpy formulation is an established method of calculating phase change problems without the need to track the phase change front. The method requires a reformulation of the heat equation in terms of enthalpy rather than temperature and so we introduced the enthalpy, H, of a system which is defined as being the sum of the sensible and latent heat of the system. Instead of thinking only in terms of the temperature of a sample we considered the energy in the system. At 100 degrees centigrade although we do not observe any temperature increase the sample continues to absorb energy. This energy is used to increase the latent heat of the water; to vapourise it. Consider water with specific heat, c_l, and density, ρ, and latent heat of vapourisation, λ being heated we find that the enthalpy, u, takes the following form

$$u = \begin{cases} c_l T & T < T_b \\ c_g T + \lambda T > T_b \end{cases} \tag{3.7}$$

where c_g is the specific heat capacity of the gas, and T_b is the boiling point of the liquid.

This describes the phase change of a liquid at a specific temperature, it is often smoothed using exponential functions in order to model a phase change over a temperature range. This smoothing over a temperature range also increases the computational efficiency of the solution. In Beckett, Mackenzie and Robertson's paper on moving mesh solutions of two dimensional Stefan problems, exponentials are used to smooth the phase transition as follows:

$$u(T) = \begin{cases} c_l T + \frac{\lambda}{2} e^{-\frac{|T-T_b|}{\epsilon}} & T < T_b \\ c_g T + \frac{\lambda}{2} - \frac{\lambda}{2} e^{-\frac{|T-T_b|}{\epsilon}} & T \geq T_b \end{cases} \tag{3.8}$$

This leads to the temperature enthalpy profile given in Figure (12)

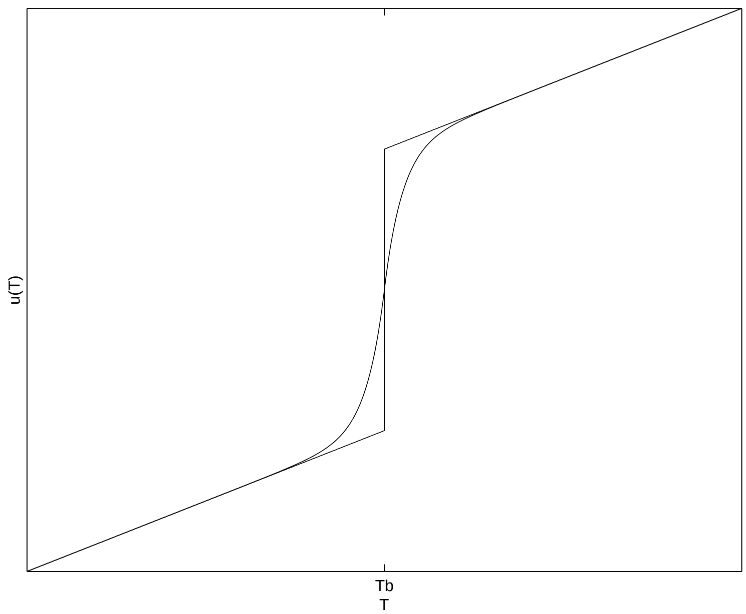

Figure 12: Temperature-Enthalpy profile with and without smoothing

When the water bound in the sample is vapourised it is then freed to escape into the oven cavity. In order to maintain the simplicity of the model it is assumed that once a region of the sample has reached 100 degrees centigrade water is boiled off and steam escapes without further affecting the system. It is further assumed that there is sufficient moisture in the system and the heating time is sufficiently low such that the food does not enter a dried phase. These assumptions meant that I did not need to model the gas phase and could concentrate the calculation to the liquid vapourisation. As a result of these assumptions the temperature of the food is limited to 100 degrees as the liquid is vapourised and the enthalpy equation becomes

$$T = \begin{cases} \frac{u}{c_l} & u \leq c_l T_b \\ T_b & u > c_l T_b \end{cases} \tag{3.9}$$

When we consider the rate of temperature change, T_t we have

$$c\rho T_t = \begin{cases} k\nabla T + P_a & T \leq T_b \\ 0 & T > T_b \end{cases} \tag{3.10}$$

As with the enthalpy method we can smooth this transition from heating to vapourising over a narrow temperature range. This smoothing increases the computational efficiency immensely without significantly reducing the accuracy of the model. We employed a hyperbolic tanh function to smooth the transition.

$$c\rho T_t = \frac{1}{2}(1 - \tanh{(\frac{T - T_b}{\tau})})\,[k\nabla T + P_a] \tag{3.11}$$

to deal with the phase change transition at the boiling point of water. This equation limits the maximum temperature the sample can attain to 100 degrees centigrade. Here τ determines the width of the phase change range. As $\tau \to 0$, we get the original enthalpy formulation without smoothing. As τ increases this increases the range of temperatures over which the phase change occurs. In the case of water and pure substances, the phase change occurs over a narrow temperature range and so for accuracy, we would like to keep τ as low as possible. However, decreasing the value of τ increases the computation time of the model. We obtained the following table for the computation time, and moisture loss after 5 minutes for the 1000W microwave oven for various values of τ

τ	Time taken	Moisture loss
0	10 Hours	37.8767g
0.01	1 Hour	37.878g
0.02	0.5 Hours	37.879g
0.05	9 minutes	37.8947g
0.1	6 minutes	40.26g

Table 1: Values of τ and corresponding time taken to calculate solution and moisture lost by the sample after 5 minutes heating in a 1000W microwave oven

It is clear that increasing τ decreases the computation time but also increases the moisture loss because we begin the phase change at a lower temperature. We find that an optimal τ of 0.05 gives good accuracy with a reasonable time.

3.9 Conclusions from the one dimensional model

- The Lambert's law approximation to the electric field intensity provided a good method of simplifying the field calculations inside the food. The approximation is valid for sufficiently large lengths such as the trays of mashed potato investigated in this project.

- The above analysis and model allowed us to examine the influence of the changing dielectric properties of the food. It was found as expected, that the field changed with decreasing moisture content and the microwaves were better able to penetrate the material resulting in a higher power deeper into the sample. However, the temperature profiles were not significantly affected by the changing field. Furthermore, the moisture content varied slightly but by not more than 7% after 5 minutes heating when compared to the constant dielectric case. This enabled us to extend the one dimensional model assuming constant dielectric properties as the above analysis implied that there was no significant effect on the temperature or moisture content of the food in taking this approximation.

- The enthalpy method provided an efficient way of dealing with the phase change at boiling. The computation time was greatly reduced without significant loss of accuracy for the appropriate value of τ, in the case of mashed potato $\tau = 0.05$ is optimal.

4 Full models for more realistic geometries , phase changes and numerical calculations

We now extend the one dimensional calculations described in the previous section to a more complete model which comprised a two dimensional model of the foodstuff with end corrections to account for the three dimensional geometry such as corner effects following conclusions in section (3.9). The two dimensional heat equation was reformulated using the enthalpy method with $\tau = 0.05$ to account for the phase change of water when the temperature reaches boiling point and Lambert's law field with constant dielectric properties. The two dimensional heat equation was then solved numerically, noting that the increase in dimension increased the size of the computation in terms of speed and memory. The previous model implemented in Matlab is used as a guide and the problem is solved using a FORTRAN program and the DDASSL stiff differential algebraic solver. For comparison with experiment we will look at mashed potato in a rectangular container in two oven designs. Using a two dimensional model with end corrections we get a reasonable agreement with experiments for a mode stirred oven then a uniform field is applied. To obtain good results in the turntable oven a varying surface field must be used. Initially we describe our work on the stirred oven.

4.1 Lambert's law in two dimensions

In a stirred domestic oven, the electromagnetic waves travel along a waveguide and before entering the oven cavity are exposed to a mode stirrer. This comprises a rotating metal vane, used to scatter the electromagnetic waves and helps to eliminate standing wave patterns in the oven cavity which lead to hot and cold spots in the food. This device is often used in conjunction with, or in place of, a rotating turntable to average out any variation in the field in the oven cavity. These mechanisms along with the internal reflections from the cavity walls, make it very difficult to calculate the electromagnetic field at any particular time. In fact, it is very difficult to measure the field distribution inside the oven cavity as the presence of a probe distorts the field. In order to quickly calculate the temperature distribution inside a heated food material quickly, it is necessary to make simplifications. To model a stirred oven we considered an averaged model for the field where it was assumed that over the period of heating the waveguides, rotations and internal reflections average out the field in the oven cavity. This model gave good comparisons with experiments.

Following the calculations in the previous sections, we simplified the complicated changing field with a simple exponential decay of electric field intensity using the Lambert's law approximation which we have shown in the stirred oven is valid for large domain lengths, (for the food investigated it is valid for domain lengths over 2cm). In order to develop a simple mathematical model for two dimensional heating in the stirred oven, it was assumed that an imposed field was applied on the boundary of a slab of food. This lead to an imposed surface power density, Q_0, at the boundary of the food. The field patterns at the surface of the food are modelled by the functions $a(y), b(y), c(x), d(x)$ a constant uniform field has $a = b = c = d = 1$. In order to model a localised reduction in the field on the upper surface (as was necessary for the turntable oven) a spatial variation such as ($d(x) = \sin^2(\omega x)$ was used. Considering each face exposed to the electromagnetic field individually Lambert's law predicts an exponential decay of the surface power density. Thus we assumed that the field within the foodstuff took the form:

$$P = Q_0(a(y)e^{-2\beta x} + b(y)e^{-2\beta(Lx-x)} + c(x)e^{-2\beta y} + d(x)e^{-2\beta(Ly-y)}) \qquad (4.1)$$

Here Q_0 represents the mean power density at the surface of the material being heated and Lx and Ly represent the length of material in the x and y axis respectively. Extending this along the length of the food leads to an approximation of the food in three dimensions, leading to an underestimation of the field intensity at the corners of the food which will experience a significant heating contribution from three faces.

4.2 Calculation of the power density

The intensity of the electromagnetic field at the surface of the material being heated was critical to developing an accurate model for the microwave heating of food. To estimate this value we calculated the power absorbed by an equivalent weight water load. This was part of a range of experiments conducted with Mr. Hooper at the CCFRA. The power absorbed by the water was calculated from the temperature rise over a fixed period of heating. Integrating the power absorbed term given above over the domain of the food gave the power absorbed by the food. From this information we were able to deduce the average surface power density.

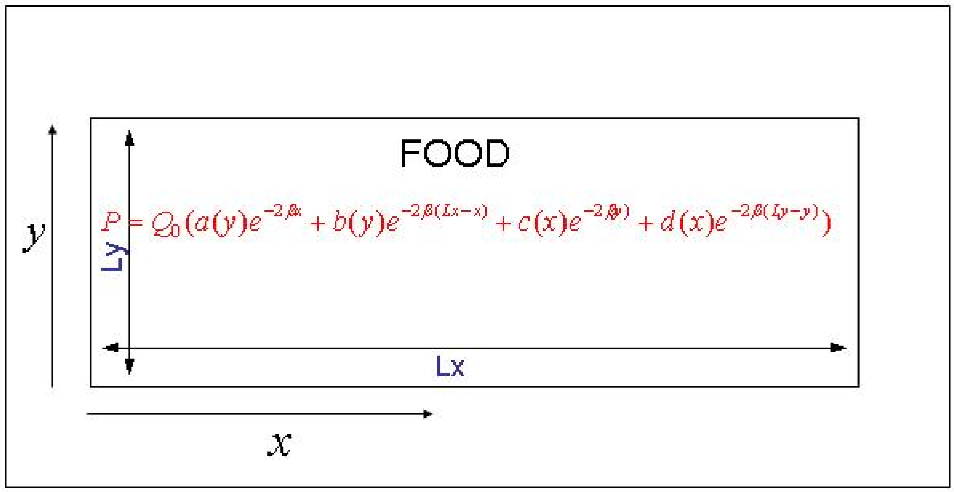

Figure 13: A diagram indicating the lengths of the sides of the model and the power absorbed term derived from Lambert's law approximation to the electric field intensity

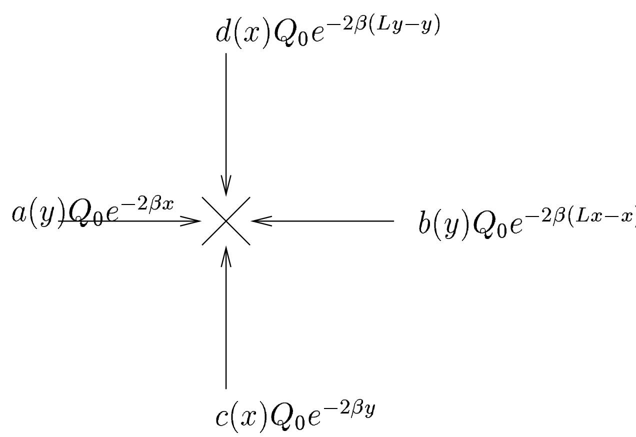

Figure 14: Heating from four sides

4.3 The heat equation in 2-D

In order to calculate the temperature in the food, we used a 2D version of the enthalpy heat equation described in the previous section (3.4). This takes the form

$$\frac{\partial u}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{Q_0}{\rho c_p}P \tag{4.2}$$

The food materials under investigation for this project have been contained in CPET plastic containers. The effects of this insulating layer were incorporated into our model by balancing the heat flux is balanced across the food plastic boundary

$$k_p\frac{\partial T}{\partial x} = k\frac{\partial T}{\partial x} \tag{4.3}$$

The plastic itself is virtually transparent to microwaves. We modelled the temperature in the plastic using the heat equation to represent the transfer of heat through conduction. In this case, the heat source arises from the heat conducted from the food to the plastic. The plastic layer is in contact with the air on the other surface. We extended the boundary conditions for the one dimensional model to include radiative heat loss as well as convective heat loss at the boundary of the plastic and food.

at $x = 0$

$$k\frac{\partial T}{\partial x} = h_c(T(t,0) - T_a) + \sigma\epsilon(T^4(t,0) - T_a^4) \tag{4.4}$$

at $x = Lx$

$$k\frac{\partial T}{\partial x} = -h_c(T(t,0) - T_a) - \sigma\epsilon(T^4(t,0) - T_a^4) \tag{4.5}$$

at $y = Ly$

$$k\frac{\partial T}{\partial y} = h_c(T(t,0) - T_a) + \sigma\epsilon(T^4(t,0) - T_a^4) \tag{4.6}$$

The final equation at $y = 0$ is a Neumann boundary condition. In the experiments conducted at the CCFRA the sample loads were placed on ceramic or glass surfaces. This boundary condition models the insulated effect of the surface.

at $y = 0$

$$\frac{\partial T}{\partial y} = 0 \tag{4.7}$$

4.4 Moisture Loss

We now investigate the calculation of the moisture lost from the sample during heating. As in the one dimensional case, the enthalpy method described in the previous section provides us with a valuable insight into the mechanism for moisture loss which can be found using (3.6). The calculation of the moisture loss can be performed at each time step which would be necessary for moisture dependent parameters as in the previous section. However, if it is assumed that the dielectric parameters of the sample remain constant with a varying moisture content we can calculate the moisture loss after the main calculation. The temperature of the sample at each time step is recorded and this data is used to extract the points in the sample that have reached boiling point. The length of time that each of these points spends at 100 degrees Centigrade, which can then be used to calculate the moisture loss when combined with the power absorbed at each point. We assumed that the water vapour is then able to leave the food without significantly affecting the load further. The assumption that the steam is free to leave the food easily means that we do not have to deal with gas transport in the food. The Greenwich model uses a sophisticated CFD package which assumes the vapour moves in a pressure driven flow through the porous medium. This model requires knowledge of the porosity of the food and also information about the relative humidity in at the food surface in the oven cavity.

4.5 Evaporation

Mathematical models for moisture evaporation and the related evaporative heat loss at temperatures below boiling exist and have been investigated as part of this project. However, it was found that there were a number of parameters in this model, the values of which were too difficult to estimate in advance. The rate of evaporation depends highly on the relative humidities inside the food, in the oven and the porosity of the sample. An investigation into the experimental data obtained at the CCFRA revealed that there is little or no moisture loss before the sample reaches 100 degrees Centigrade and by far the dominant mode of heat loss was the boiling of water in the food. Hence we did not pursue our work on evaporative moisture loss.

4.6 End Corrections

In the microwave heating of food the majority of heating occurs in a boundary layer at the surface of the food. The depth of this is influenced by the penetration depth of the food. The food experiences the most rapid heating at the corners and edges of the food where the field can penetrate most easily and be most concentrated. This is demonstrated in both the two dimensional model and the thermal images. The two dimensional model presented here was unable to model the high fields experienced at the corners and ends of the sample. This is a limitation of modelling the three dimensional heating process with a two dimensional model. In order to compensate for the high fields experienced at the ends of the sample, we used an end correction to allow for the special effects of the edge heating. The two dimensional model represented a cross section through the mid section of the food. This gives an idea of the temperature distribution throughout the majority of the sample. However, the ends of the sample experience a greater heating effect than the centre cross section. The end correction used in the numerical calculation assumes that the fields observed at the ends of the sample are similar to the field observed by the sides of the vertical cross section. These side areas experience high field intensities from two faces. This end correction deals with the ends of the sample but the corners of the sample are still approximated by a field from two sides, when in reality they experience a field from 3 sides. This leads to an underestimation of the temperature increase and hence moisture loss experienced at the corners.

We were able to perform a simple calculation to determine the approximate moisture loss from the corners of the load. This is then added to the moisture loss profiles produce as above to account for the moisture lost at these points.

4.7 The turntable oven

The model for the stirred oven assumed an averaged field using a constant surface power density. This assumption implied that the field was uniform around the food over the duration of heating owing to the effects of scattering. In turntable ovens the food is rotated through the field as there is no scattering this field can, and does, set up standing wave patterns. This can lead to significant field effects manifesting themselves as hot and cold spots in the food. In the thermal image figure 16 we see that after 5 minutes heating there is a region food in the centre of the load significantly cooler than the rest of the sample. These thermal images were taken after several minutes heating in a 750W turntable oven.

The load was placed at the centre of the turntable. From the plot in figure 16 it appears that there was a minimum in the field at the centre of the oven and the food was rotated around this minimum position, accounting for the region of lower temperature evident in the thermal image. Evidently in this case the uniform field assumption breaks down and must be replaced by a variable surface field. A simple modification of Lambert's law allows us to account for such a variation through appropriate functions a, b, c, d. It was assumed that there was little variation in the field from the sides and so we chose $a(y) = b(y) = 1$. The thermal image above indicated little heating from below which fixed $c(x) = 0$. In order to model the reduced top heating and retain the high fields experienced at

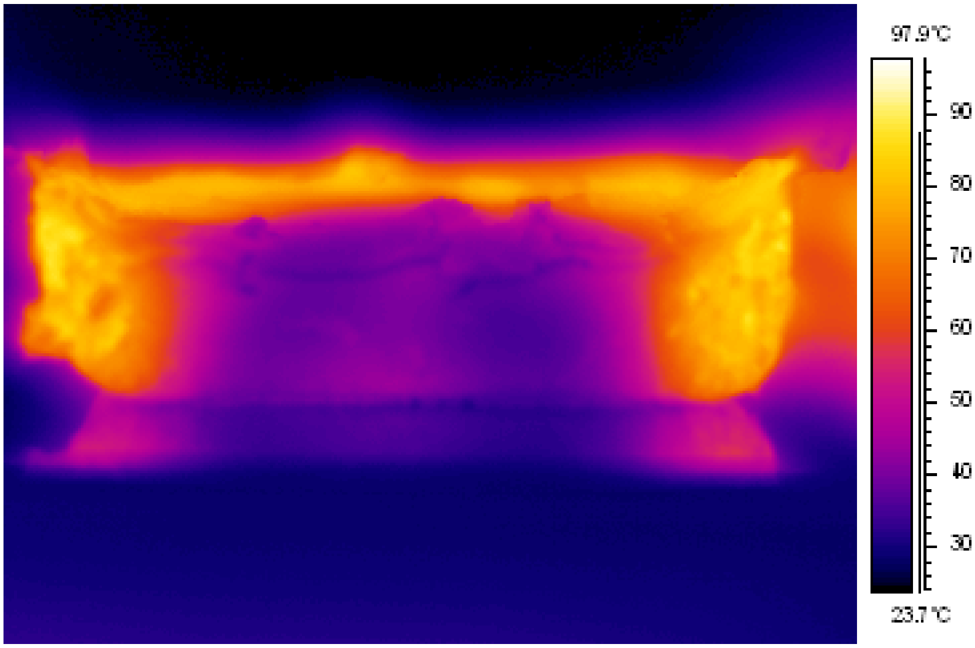

Figure 15: Thermal cross section of a sample of mashed potato after 3 minutes heating in a turntable microwave oven

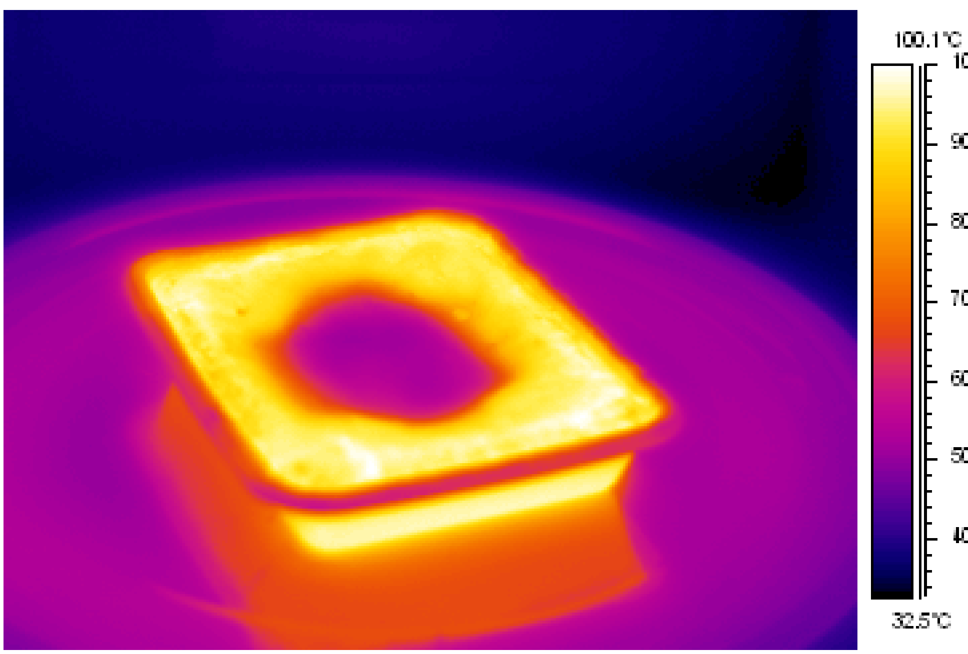

Figure 16: Thermal image of a tray of mashed potato taken after 5 minutes heating in a 750W turntable oven

corners the following function was chosen $d(x) = \sin^2(\pi x/Lx)$ which is equal to 1 at the edges and 0 at the centre of the upper face. This approximation allowed us to model the effects of hot and cold spots in the food without solving Maxwell's equations. Although the model is not able to predict the presence of hot and cold spots, if they are known they can be incorporated into the model. The Greenwich model and the work by Metaxas discussed in section (2.3) calculate the field inside the oven cavity and so are able to predict the field patterns on the food. This information can be used in our model to approximate the field patterns using the functions described above.

4.8 Numerical Calculations

The temperature profile and moisture loss of a sample of food heated in a microwave oven were found by solving the model described numerically in equations (4.2)-(4.7). The solution to the equations was found using a program written in the FORTRAN computer programming language. This was used as it has a number of efficient numerical solvers and the code runs quickly on a desktop machine. The model required several input parameters relating to the dielectric and physical characteristics of the food and the power absorbed by an equivalent load. In particular the code required the following input parameters, the specific heat capacity, density, thermal conductivity, dielectric loss and the dielectric constant of the food. The dimensions of the rectangular tray were also needed along with the power absorbed by an equivalent load. These values were then used to calculate the temperature distribution along a vertical cross section of the food. The temperature data was then used to calculate the moisture loss of the sample throughout the heating time. The one dimensional analysis was used to estimate the validity of using the Lambert's law approximation to the electric field. The schematic in Figure 17 describes the use of the code.

The equations were solved numerically by making use of the finite difference disctretisation in space to give a system of differential and algebraic equations. This system of equations was the solved using the stiff ODE solver DDASSL in FORTRAN, which uses a BDF method for calculating the solution vector.

The model is able to predict the temperature at the centre line of the food load and the overall moisture lost by the sample. We are not able to predict the temperatures at the corners of the food. The moisture lost is calculated for the whole sample and not local moisture losses. The model does not calculate the moisture transport within the food as it assumes that there is little water movement in the food and that the water vapour escapes without significantly affecting the food. Initial experiments conducted at the CCFRA found little evidence for moisture movement in the samples of mashed potato.

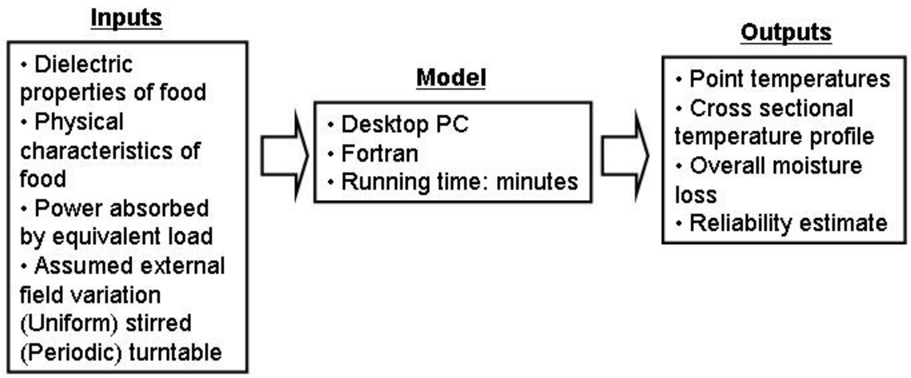

Figure 17: Diagram indicating the use of the computer code developed to solve the model for microwave heating

5 Results and Experiments

The model described was developed with constant reference to experimental data and practical experience. This information gave us further insight into the main physical process involved and allows us to verify the model and numerical calculations. In this section, we outline the experiments conducted at the CCFRA. We present a comparison of the numerical solution to the model derived in the last chapter with the data gathered from trials.

5.1 Methods and Measurements

Initial experiments conducted by Mr. Hooper at the CCFRA to gather temperature and moisture loss data for microwave cooking used samples of mashed potato in rectangular CPET plastic trays. Mashed potato was chosen as it is an easily reproducible mixture to form. More importantly it is a relatively homogeneous material which is malleable. Since the mashed potato mixture was initially dry it was possible to record the amount of water added to the mixture to give a percentage moisture content for the sample. This is of particular interest in the calculation of the dielectric properties in the Greenwich model which uses an homogenised model of the dielectric properties of each phase of the material. The plastic trays used were virtually transparent to microwaves, and so did not significantly alter the absorption of the electric field by the food. There are many different makes and models of microwave oven in everyday use. Domestic microwave ovens are classified by their power rating. In the experiments conducted at the CCFRA three ovens were used of different power, two with a mode stirrer device and one with a turntable.

5.2 Measurements

The equipment at the CCFRA allowed us to monitor several aspects of the heating process. In general we measured the temperature of the samples continuously and we could also easily measure the weight at times 1-5 minutes. The surface temperature of the heated sample was measured using a thermal imaging camera. Using post processing software, these images can give spot temperatures, average temperatures and line temperatures. The thermal images produced provided a valuable insight into the variations that occurred in the field inside the oven. The hot and cold spots which occurred on the surface of the heated load shown in figures 15 and 16 were evidence of the standing wave patterns set up in the oven cavity. The thermal imaging camera is unable to give and accurate temperature reading during heating as the camera cannot resolve the temperature of the heated sample through the oven door. In order to gain temperature information during the heating process, we made use of fibre optic thermometry. Fibre optic probes in the sample allowed us to monitor the temperature change at a particular point in the sample continuously during heating. The fibre optic probes have the advantage that they have a negligible effect on the electric field inside the oven and so do not significantly affect the temperature rise. The probes were connected to a PC which recorded and plotted the temperature at each probe location at all time values. The connection to a PC means that without a counter rotating oven we cannot measure the temperature using a turntable oven and hence the majority of the measurements were made for mode stirred ovens. The overall moisture lost by the sample was found by recording the weight of the sample before and after heating. As the water bound within the food is vapourised, it escapes via the surface of the food as steam. This leads to a weight loss in the sample together with a small loss due to evaporation. By heating a new sample for each run for several heating different heating times, we were able to see how the moisture content of the sample changes. The calibration of the model described in the previous section required the average power absorbed by the sample as an input. In order to get an accurate figure for the power absorbed by the sample, a simple water load test was used. A water load equivalent to that of the food load was placed in the oven at room temperature. After a set time of heating the temperature rise in the water load was measured. From the heating time and the temperature rise we were able to deduce the power absorbed by the water load. This value was then used to calculate the surface power density, Q_0 in the formulae (4.1).

5.3 Methods

The samples used in the following experiments were Smash instant mashed potato. The Smash was formed from a mixture of 88g dry mixture, 488g water. This was mixed until the mash formed a homogeneous consistency. The mixture was then used to fill rectangular CPET trays to a weight of 300g. This weight also corresponds to a depth of 2cm in the trays used. The surface of the samples were then smoothed to ensure an even interface. The samples were covered in a polythene film and placed in a refrigerated area overnight. The film was used to minimise any moisture lost during the chilling process. The overnight chilling process ensured that the samples were at a uniform temperature of 5 degrees Centigrade throughout the sample. When conducting the trials the following day, the samples were collected from the chiller is small batches and kept in a domestic refrigerator. The time between removing the sample from the refrigerator and heating was kept to a minimum.

Once removed from the refrigerator the samples were immediately weighed, this weight was used to find the overall moisture lost during the heating process. The tray was then placed into the centre of the oven and the probes were inserted at the required location. The probes were held in place by means of a plastic holder. The probes were threaded through small holes in the plastic and held in place by rubber gromets in the holes. The probes were then set to the appropriate depth. The plastic holder sat on top of the plastic tray and so could easily be placed in the required position and removed quickly.

The thermal probes were connected to a PC via a control box. The temperature in the microwave was recorded through four Fluxtron thermal probes. The computer began recording the temperature a short while before the microwave began heating to ensure the whole heating process was captured. Once the heating regime finished, the door was opened and a thermal image taken using a FLIR thermal imaging camera. The probes were then carefully removed and the sample was weighed.

We measured the temperature continuously using fibre optic thermal probes. These were placed in the sample at a depth of 10mm along the midsection of the tray in the configuration in Figure 18.

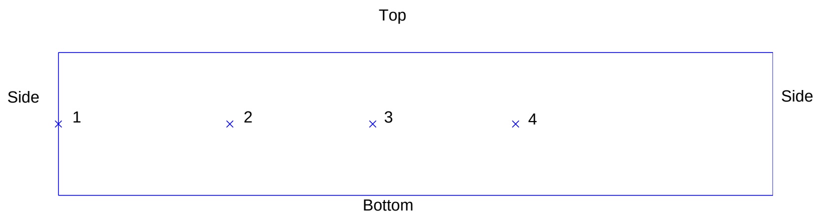

Figure 18: Probe locations in the sample

5.3.1 Mode stirrer Ovens

We began our investigation looking at microwave ovens with mode stirrers. The lack of a turntable allowed for the use of fibre optic probes to give information about the temperature inside the food load. Looking at a thermal image of the cross section of the mashed potato after 3 minutes heating, Figure 19, we see that the heating effect inside the load is very even, this leads us to conclude that there is a heating effect from all sides of the material.

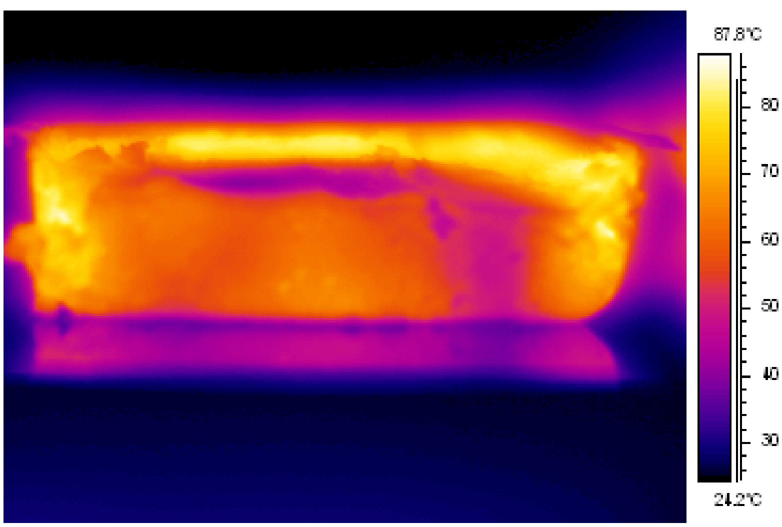

Figure 19: Thermal cross section of mashed potato heated in a microwave oven with a mode stirrer for three minutes

5.4 1000W mode stirred oven

We began by comparing the results of our numerical calculations with the experiments conducted using a 1000W oven. The power output experiments on a 300g water load gave an average power dump of 730W. This is used to calculate the surface power density. The power rating of the oven and the power absorbed by the load have a large discrepancy of 250W. This is most likely due to field effects within the oven. This can lead to uneven heating patterns in the oven which may account for the power difference and the variation in results. The model predicts the temperature profiles given in Figure 20 for 1 to 5 minutes heating.

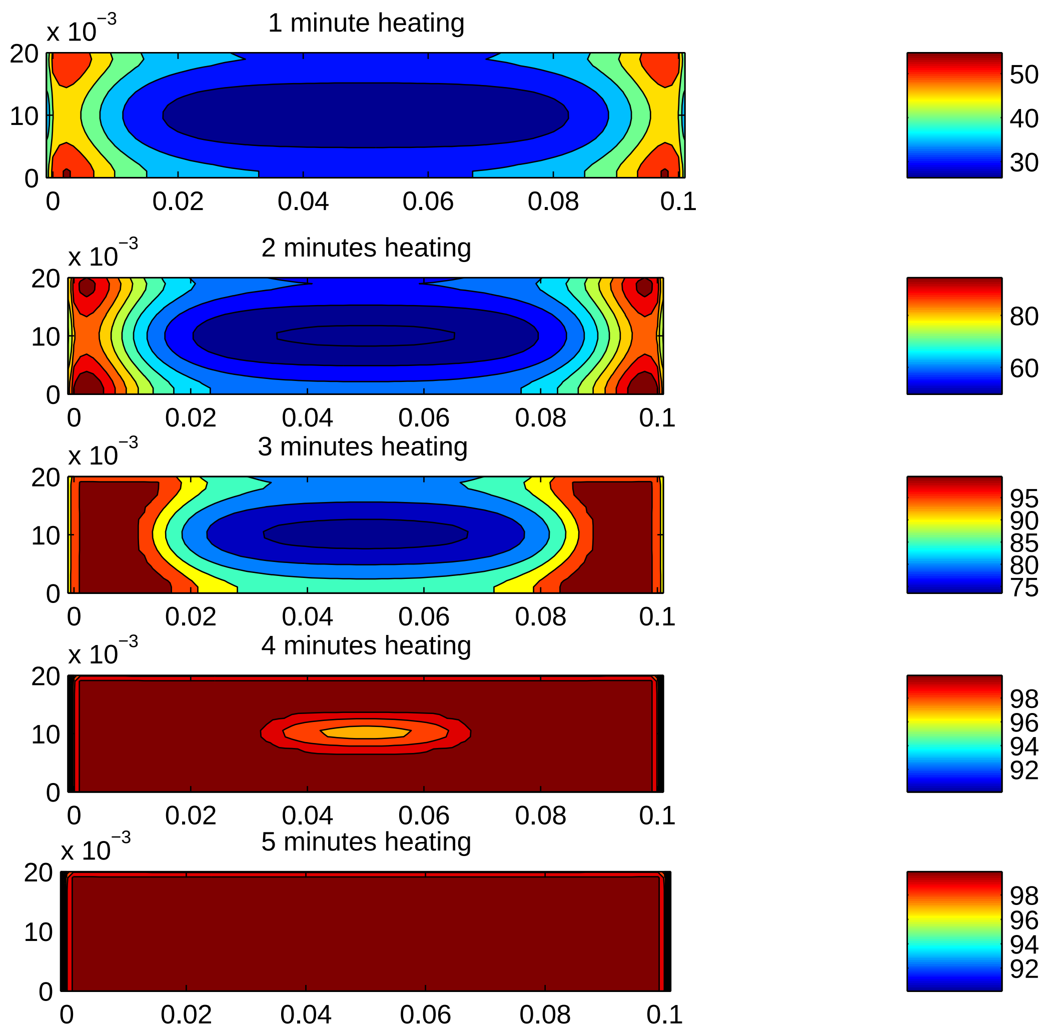

Figure 20: Contour plots of the temperature in sample of mashed potato in a 1000W oven. Heating times are 1min, 2min, 3min, 4min and 5 minutes

The attenuation for the field is clear from the cross sections, the corners experience the highest field intensity which consequently leads to a higher rate of temperature increase. The edges of the sample also heat rapidly and this leads to a front of 100 degrees centigrade which moves into the centre of the sample. We can compare this data with the cross sectional thermal images earlier and also the thermal image, Figure 21 of the upper surface of the food after 5 minutes heating.

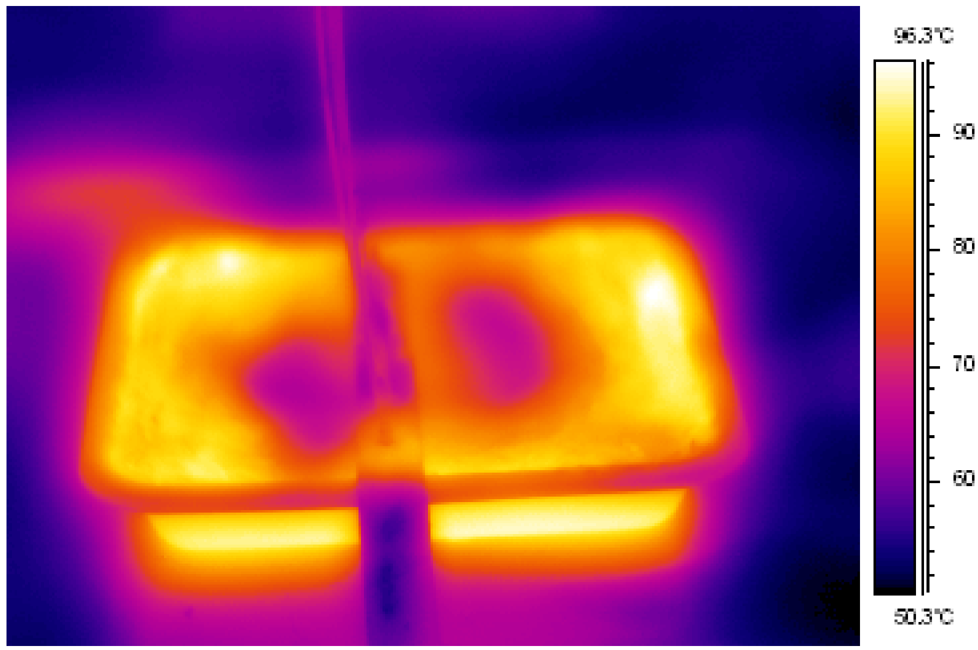

Figure 21: Thermal image of a tray of mashed potato after 5 minutes heating in a 1000W mode stirred microwave oven

There appears to be a region in the centre of this sample which has reached a lower temperature than the rest of the sample. However, looking at the cross sectional thermal image above, it seems as though this is a surface effect. In the cross sectional thermal image the centre of the food is slightly hotter than the surface. This is possibly the result of evaporative cooling. The steam leaving the food has a cooling effect on the external surface.

We then compared the point temperatures as predicted by the model with the fibre optic thermal probe data. The experimental data is gathered for several heating times needed to compile a moisture loss history and is plotted in solid blue lines. The numerical prediction for the temperature rise is given as a dashed red line in figure 22:

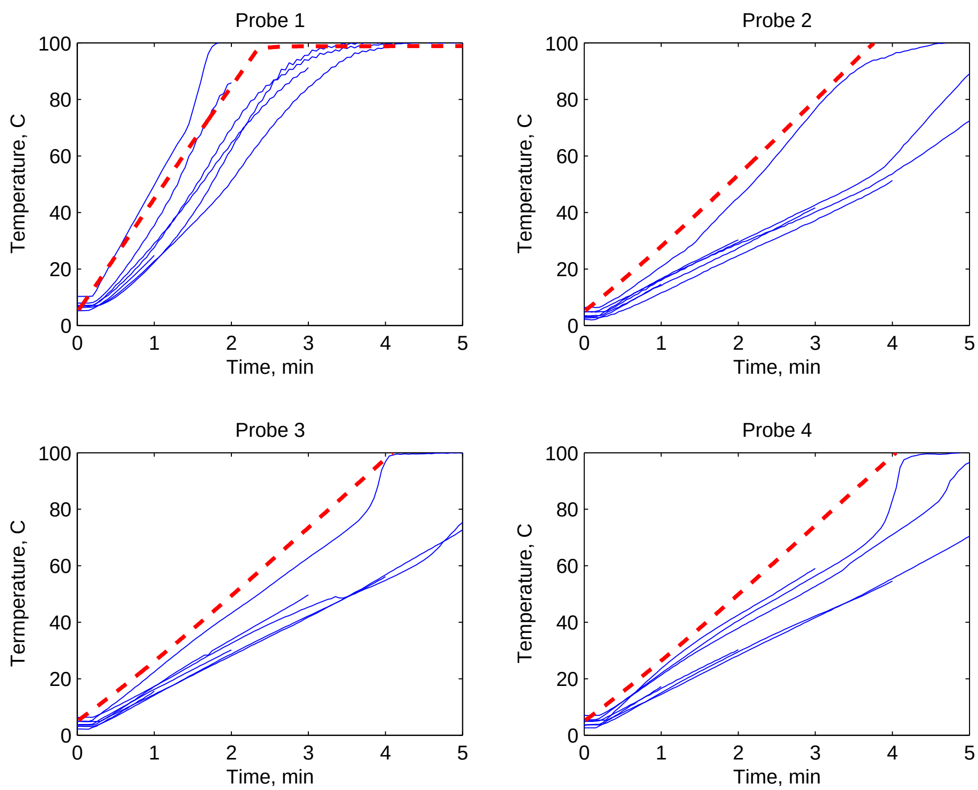

Figure 22: Comparison of experimental thermal probe data in solid blue with numerical model prediction in red dashed lines for 1000W oven

Probe number 1 is placed on the edge of the sample. This probe experiences the highest field intensity and we would expect the highest temperature rise. The computed temperature is slightly higher than the experiments. The temperature rises rapidly until the sample reaches 100 degrees Centigrade. We see good agreement between the experiment and numerical calculations. The experimental results were gathered from trials of 1-5 minutes heating. After each sample was heated the probes were removed and placed in a new sample, this made it difficult to place to probes in exactly the same position which would account for some of the variation the in the experiments.

The moisture lost by the sample is measured by the weight loss over the course of the heating. The moisture loss is calculated in the mathematical model by assuming that water at 100 degrees boils. Comparing the experimental result with the numerical model gave the Figure 23

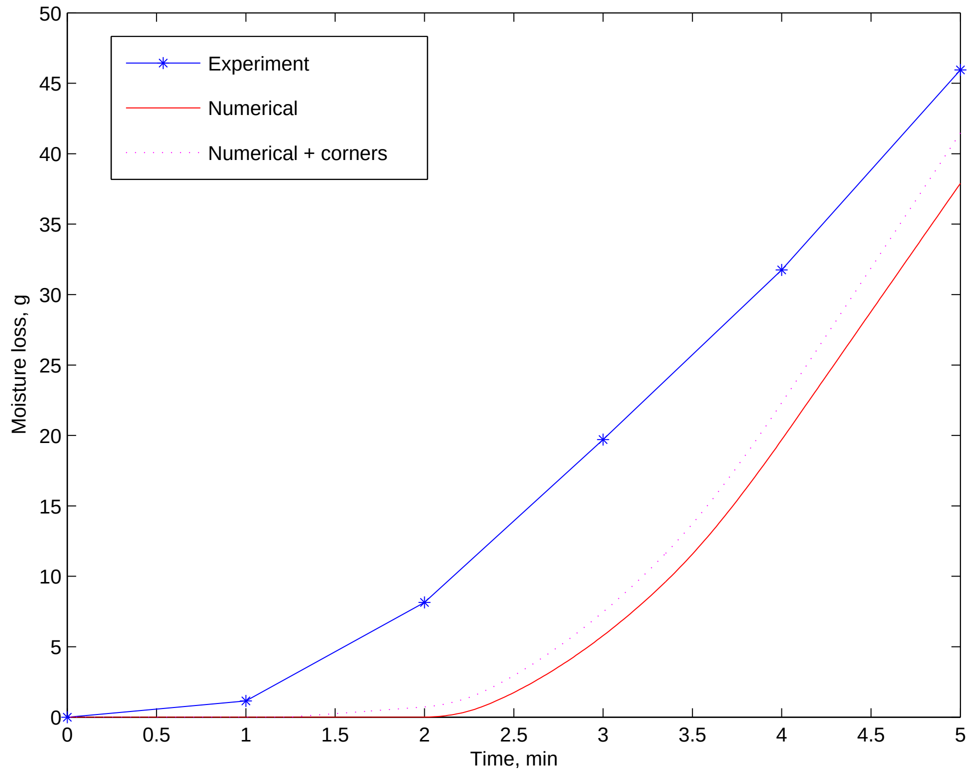

Figure 23: Moisture loss curves for 1000W oven

Heating time, min	1	2	3	4	5
Run 1	1.3g	8.5g	21g	32g	45g
Run 2	1g	7.8g	18.4g	31.5g	46.9g
Average	1.15g	8.15g	19.7g	31.75g	45.95g

Table 2: Moisture lost in each run

The numerical calculations in solid line compares reasonably well with the experiments. The numerical calculation underestimates the moisture loss. The model does not begin to lose moisture until 1.5 minutes. The experimental data suggests that the sample begins losing moisture before 1 minute. This should be expected as the model does not deal with the corners of the sample accurately nor does it model initial evaporative moisture loss. In experiments we expect the corners to experience the highest fields and so the most moisture loss. The dashed line represents the computed moisture loss with the effect of rapid corner heating included.

We can also investigate the effects of varying the field in the sample. In the following calculation it was assumed that there was no heating effect from below but that the same total power was absorbed, this resulted in a higher field intensity on the remaining three faces and subsequently a higher moisture loss. The point temperatures along the midsection were unaffected, the moisture loss profile, Figure 24 was found

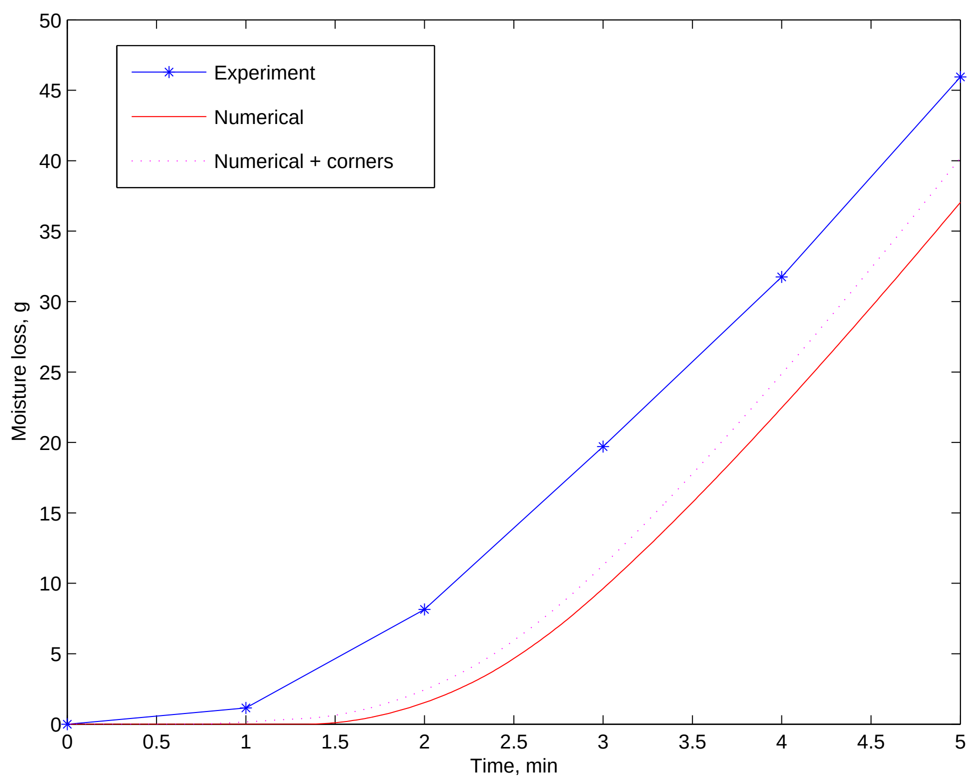

Figure 24: Moisture loss curves for 1000W oven with no heating from below

The model with no heating from below gives a closer approximation to the experimental results and also matches the rate of moisture loss (the curves are parallel). This indicates that there is probably a lower field intensity on the lower surface of the food. The Bath model is unable to predict this, however in conjunction with the Greenwich team's simulation it is hoped that the field around the food can be found. The initial moisture loss is still lower than the experiments which indicates that there is some moisture loss in the early stages of heating which is not captured in the model.

5.5 650W mode stirred oven

Experiments were carried out on a mode stirred microwave oven with a different power rating. The following experiments were conducted using a 650W microwave oven. The power output experiments on a 300g water load give an average power dump of 615W. This is then used to calculate the surface power density. The model predicts the temperature profiles for 1 to 5 minutes heating given in Figure 25.

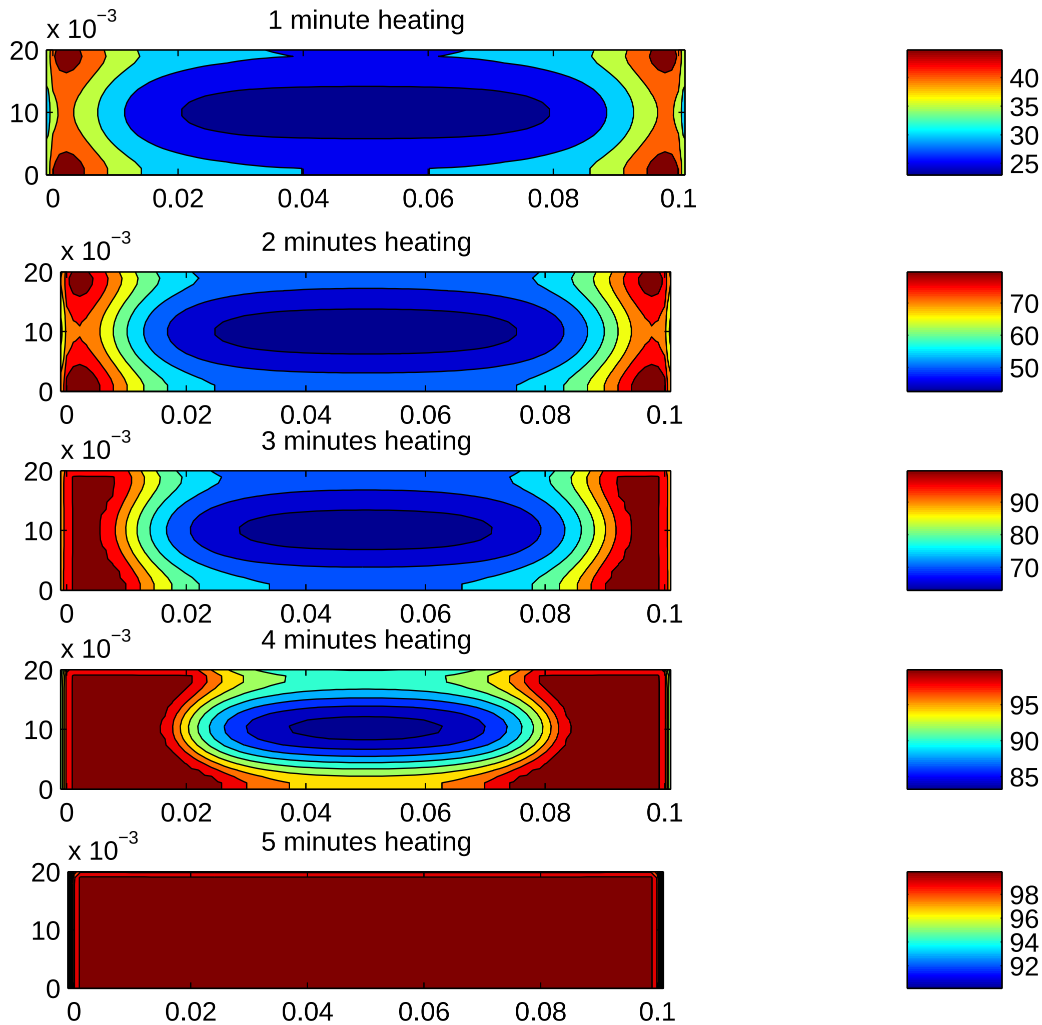

Figure 25: Contour plots of the temperature in sample of mashed potato in a 650W oven. Heating times are 1min, 2min, 3min, 4min and 5 minutes

In the above calculation we see a similar pattern of heating as in the 1000W oven. We compared the above results with the thermal image of the upper surface taken after 5 minutes heating in Figure 26.

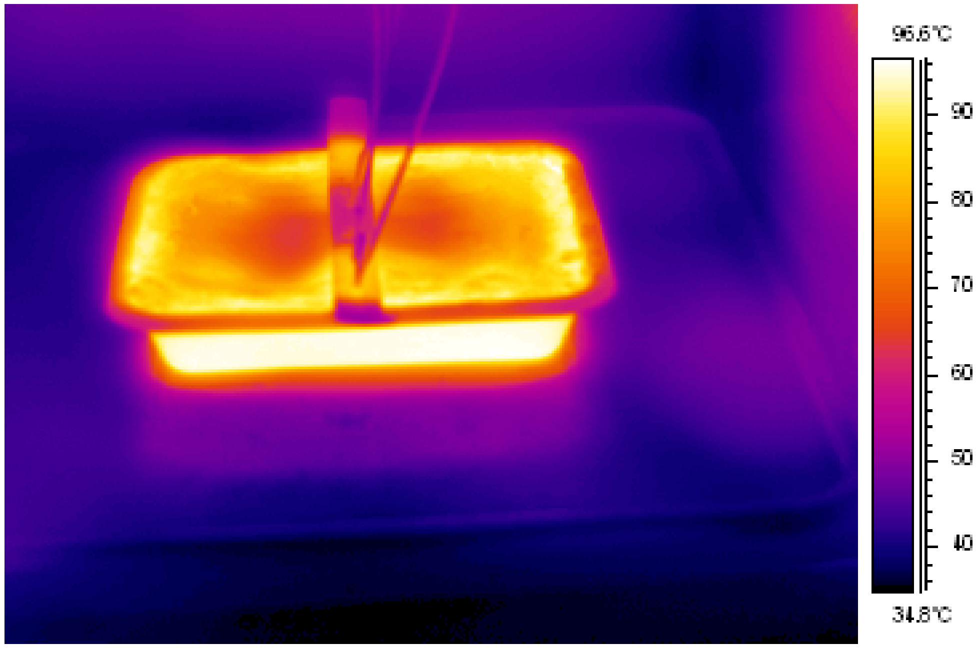

Figure 26: Thermal image of a tray of mashed potato after 5 minutes heating in a 650W mode stirred microwave oven showing near uniform surface temperatures

The sample appears on the surface to have reached a uniform temperature close to 100 degrees centigrade. This is predicted by the mathematical model. It can clearly be seen in the numerical cross sections that after 5 minutes heating the entire sample has attained a temperature of 100 degrees. The thermal image shows the location of the probes as well as the plastic holder to keep the probes in a fixed position.

The point temperatures predicted using the mathematical model were then compared with the fibre optic thermal probe data. The numerical prediction for the temperature rise is given as a dashed red line in Figure 27

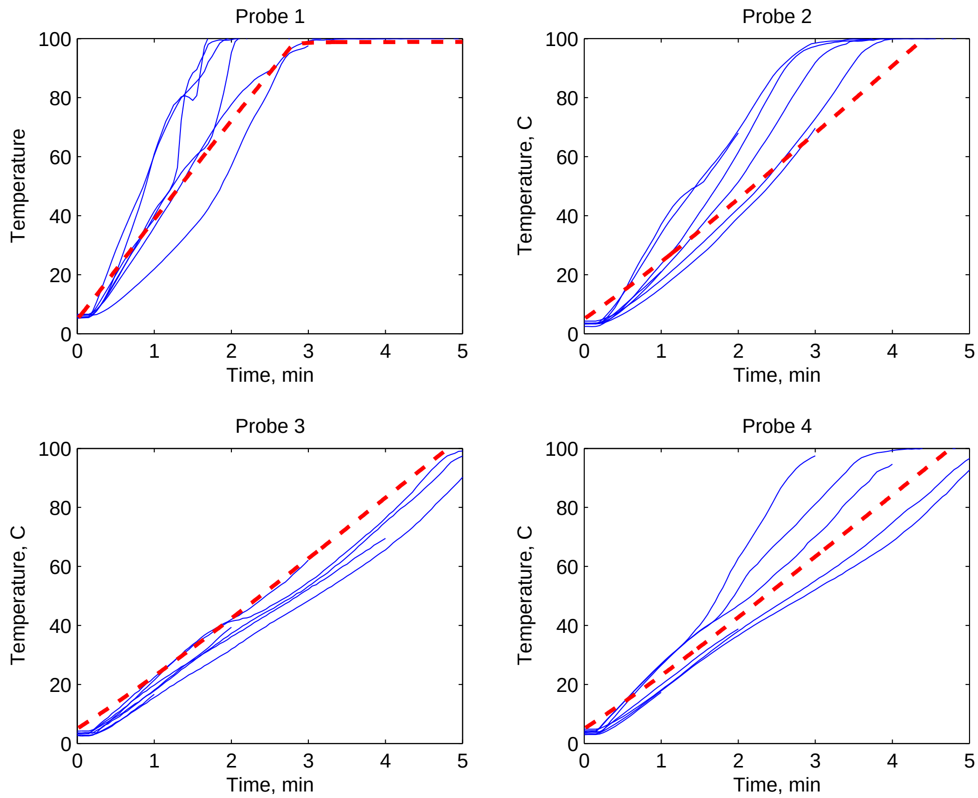

Figure 27: Comparison of experimental thermal probe data in solid blue with numerical model prediction in red dashed lines for 650W oven

The numerical calculations gave a closer agreement to the thermal probes for the 650W oven than the 1000W oven previously. The power rating of the oven and the power absorbed by an equivalent load appear to be very similar. This indicates that the majority of the power entering the oven cavity is absorbed by the load.

The weight loss for each of the experiments was recorded to give an idea of the overall moisture loss of the sample over the heating period. This was then compared with the numerical calculations as in the previous oven Figure 28.

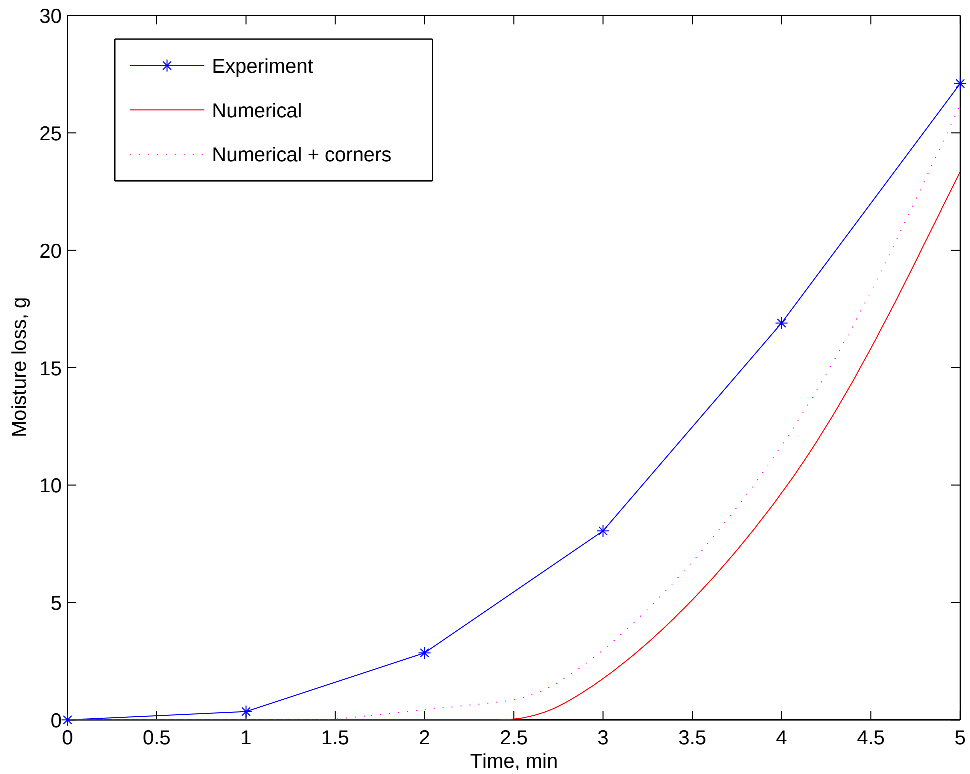

Figure 28: Moisture loss curves

Heating time, min	1	2	3	4	5
Run 1	0.4g	3.4g	9g	18.6g	30.2g
Run 2	0.3g	2.3g	7.1g	15.2g	24g
Average	0.35g	2.85g	8.05g	16.9g	27.1g

Table 3: Moisture lost in each run

It is clear that the numerical model again underestimates the initial moisture loss, however the results have a better agreement than for the previous 1000W oven. The added corner correction increases the moisture loss at the end of the heating run.

We also investigated the effect of heating from below on the 650W microwave oven. The moisture loss was again found to increase without significantly affecting the point temperature profiles Figure 29

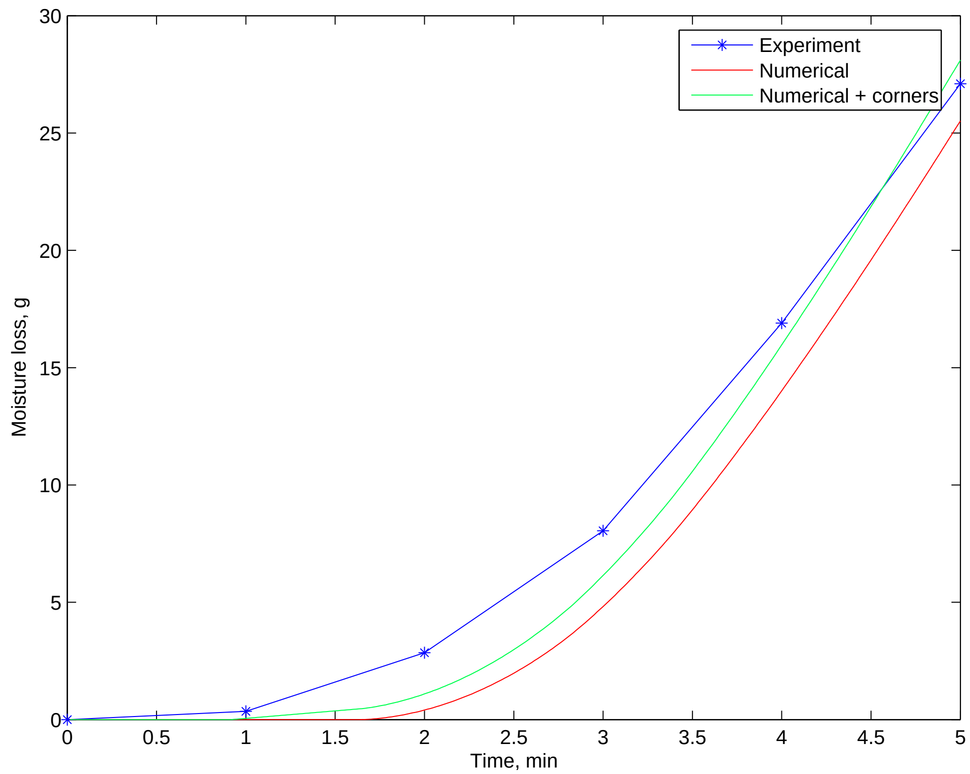

Figure 29: Moisture loss curves for 650W oven assuming no heating from below

The model with no heating from below gives a much better agreement with the experimental values with the rate of moisture loss. As in the 1000W oven case the moisture loss predicted by the model underestimates the experimental values, again possibly due to evaporative loss.

5.6 750W oven turntable oven

We now consider experimental results for a 750W turntable oven. The aim of the experiments was to gain thermal and moisture data of the sample of mashed potato in a waveguide oven with a turntable and no mode stirrer. The use of a turntable prohibited the use of thermal probes in the food. In the earlier section (4.7) the field variation was observed particularly in the thermal image of the upper surface after 5 minutes heating given in Figure 16. The assumed dip in the field was modelled as in section (4.7) and the temperature profiles over 5 minutes heating were found and are given in Figure 30.

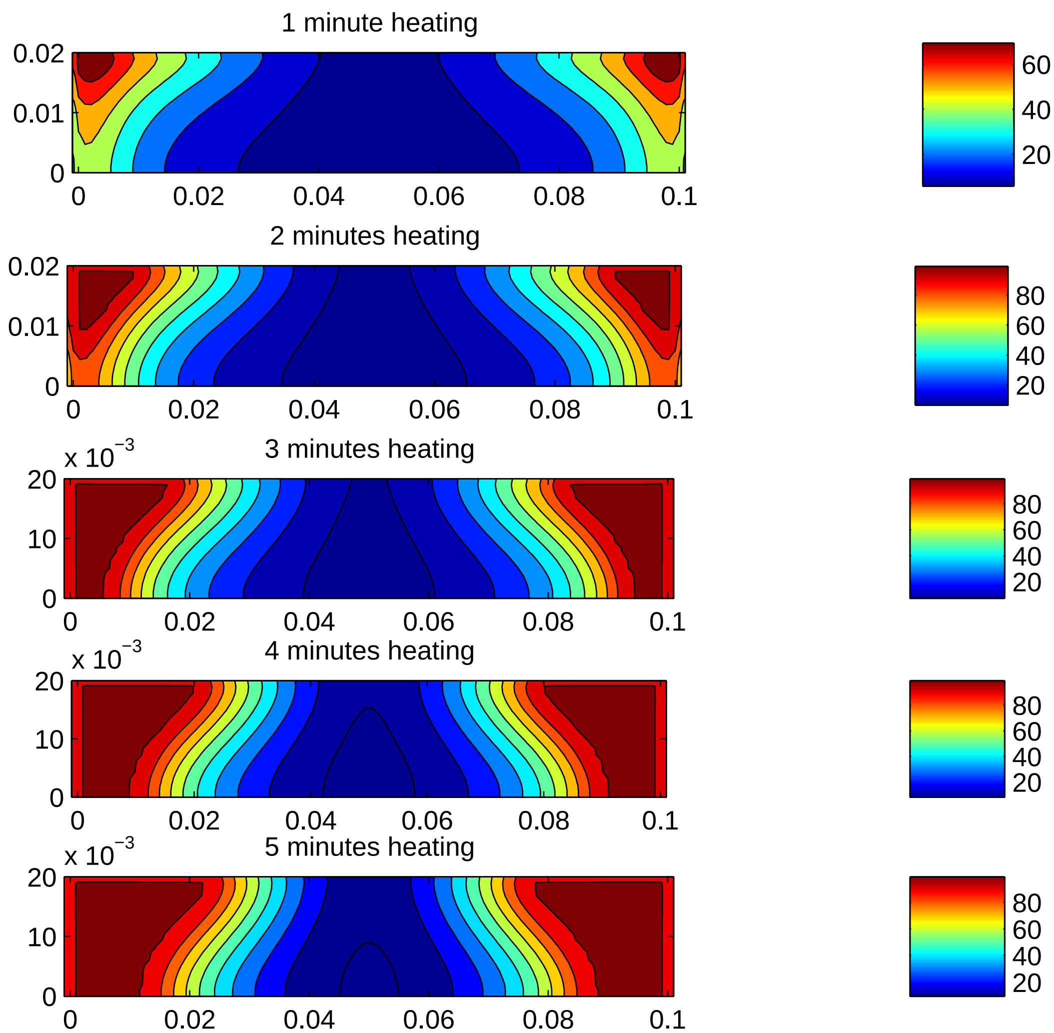

Figure 30: Contour plots of the temperature in sample of mashed potato in a 750W oven. Heating times are 1min, 2min, 3min, 4min and 5 minutes

The approximation to the field patterns resulted in the hole in the centre on the upper surface and the lack of heat generation on the lower surface.

As in the previous oven, the weight of the samples before and after heating was used to estimate the moisture lost by the sample. The numerical model described above revealed the moisture loss curves as given in Figure 31.

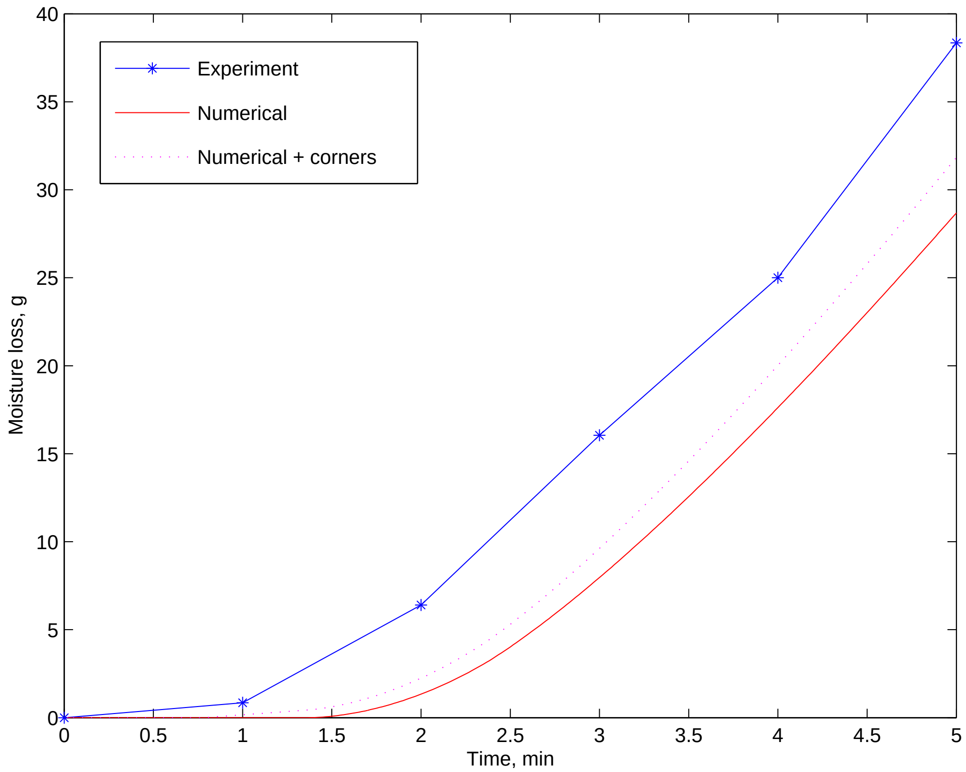

Figure 31: Moisture loss curves for the turntable oven

Heating time, min	1	2	3	4	5
Run 1	0.8g	7.2g	16.2g	25.1g	39g
Run 2	0.9g	5.6g	15.9g	24.9g	37.7g
Average	0.85g	6.4g	16.05g	25g	38.35g

Table 4: Moisture lost in each run

As in the previous examples the model initially underestimates the moisture lost by the sample. In the subsequent heating the rate of moisture lost by the sample is accurately predicted by the model.

5.7 Greenwich calculations

The above comparisons between experiment and our model show a good agreement for the temperature and overall moisture loss. However, from the turntable oven experiments it became apparent that some knowledge of the field patterns is required to accurately model the heating process. In the above experiments the field pattern is deduced from the thermal images. The Greenwich model is able to calculate the field patterns both in the oven cavity and within the food. This information would allow us to calibrate the Bath model's field pattern.

6 Discussion and areas of further investigation

We have developed a simple model for the temperature evolution and the moisture loss of a sample of chilled food heated in a microwave oven. The electric field was approximated in two dimensions using the Lambert's law approximation. This approximation was validated using analysis on one dimensional models where it was found that for sufficiently large lengths the approximation was close to the exact solution from Maxwell's equations. Variable fields were found to occur in turntable ovens and these were modelled by assuming an imposed surface power density. The model was first used to calculate the temperature distribution in a vertical cross section of a rectangular tray of mashed potato and then to calculate the moisture lost by the sample throughout the heating process. The moisture loss, found numerically, was found to underestimate the experimental values. This was found to be, in part, due to end and corner effects. The end effects were incorporated into the model however the corner effects are the result of field patterns in three dimensions and were not included in the model. The power absorbed was calculated using the dielectric properties of the food. After careful analysis on a one dimensional model, it was found that over short heating times (of less than five minutes) the changes in dielectric properties do not appear to have a significant influence on the temperature or moisture loss of the sample.

6.1 Model Strengths

The model gave good agreement with the experiments on samples of mashed potato in rectangular trays. The point temperatures taken midway into the sample were found to lie close to the predicted temperatures. The moisture loss was found to be sensitive to the imposed field but still gave a reasonable estimate of the moisture loss over the heating time. The cross sectional temperatures appear to be consistent with the thermal images provided that a uniform field is assumed for a mode stirred oven and a sinusoidal field is assumed for the turntable oven The model has shown good average predictions in a calculation time of only a few minutes on a desktop machine.

6.2 Model Weaknesses

The model was not able to calculate the moisture lost in the early stages of heating, this lead to an underestimation of the overall moisture loss. The model does not take into account evaporative losses and assumes a very simple model for moisture escape. The model is unable to predict the temperatures throughout the entire sample as corner effects become prominent and these are not included in the model in the interests of maintaining a low computation time. The calculations require an estimation of the external field to account for variations such as are seen in the turntable oven.

6.3 Extension

Work is currently underway in linking the results from simulations conducted using the University of Greenwich's electromagnetic solver with the Bath model. The resulting field from the simulation can be used to calculate an assumed surface power density. This would greatly improve the Bath model's ability to model the variation in the field inside the oven cavity. The model developed at Bath university has been used to predict the temperature and moisture loss for the turntable oven and the results compared with experiments. The experiments revealed that there appears to be some standing wave patterns in the oven cavity which are not averaged out through rotation. The work currently underway aims to simulate the turntable oven heating using the Greenwich calculation to obtain a better understanding of the field and temperature distributions inside the food. The three dimensional simulation of the microwave oven will allow us to gain a greater understanding and reveal the field patterns within the oven cavity and within the food load.

As mentioned earlier the model does not include evaporative loss, this could account for the initial underestimation of the moisture loss. In the initial investigation it was found that there are a number of parameters which would be difficult to obtain to accurately model the evaporation process.

An important aspect of domestic microwave cooking is the heating of food from frozen. This phase change is not included in our model. The enthalpy method provides an excellent way of dealing with phase changes and so the thawing of food can be included in the model. This would require temperature dependent dielectric properties because of the vastly different properties of ice and water.

A Thesis Outline

1. Introduction
2. Models of microwave heating
3. One dimensional analysis of Lambert's Law and Maxwell's equations
4. Heating effects, enthalpy method, variable dielectric properties compared with constant dielectric properties
5. Two dimensional models, field at boundary (Greenwich), end corrections and numerical methods
6. Experimental results
7. Discussion
8. Bibliography

B Poster presented at the Faraday day, Oxford, 21st November 2005

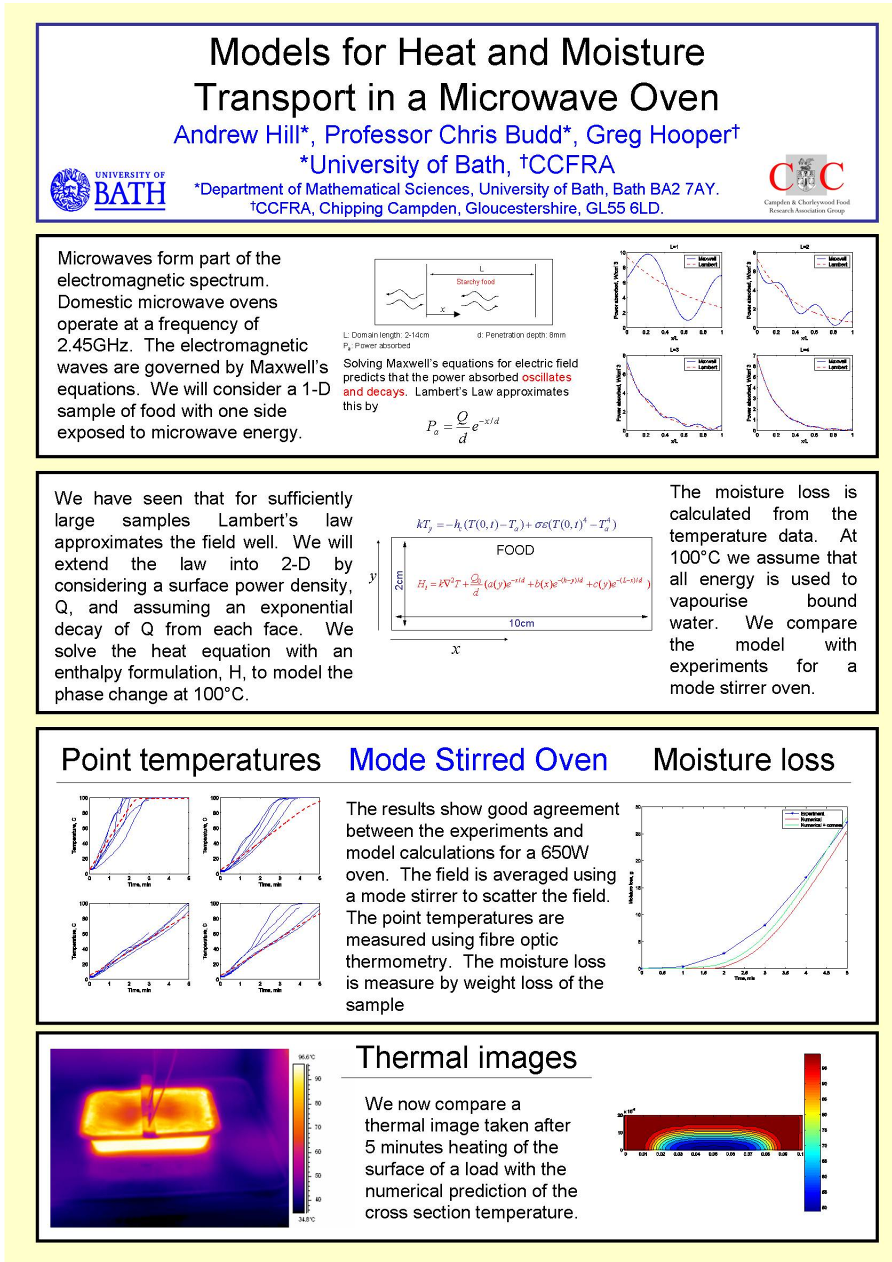

Figure 32: Poster presented at the Faraday day at St. Catherines College, Oxford on the 21st of November 2005

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